ELSEVIER

Physica E 6 (2000) 301–305

PHYSICA E

www.elsevier.nl/locate/physe

Entanglement in 2DEG systems: towards a detection loophole-free test of Bell's inequality

Xavier Maître*, William D. Oliver, Yoshihisa Yamamoto

Quantum Entanglement Project, ICORP, JST, Edward L. Ginzton Laboratory, Stanford University, Stanford, CA 94305-8045, USA

Abstract

We make use of the intrinsic noiseless and lossless aspect of an electron source in a 2DEG system to implement a new test of Bell's inequality. The generated entanglement can be tested by two-particle interferometry. Preparation and detection schemes of two complete sets of Bell states are given. A novel type of Bell's inequality is then derived in terms of noise correlation measurements. The characteristics of the electron source are essential to exhibit a violation. This electron system could close the detection efficiency loophole. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 03.65.Bz; 05.30.Fk; 73.40.Hm

Keywords: Two-dimensional electron gas; Two-particle interferometry; Bell states; Bell's inequality

1. Introduction

The indistinguishability of two identical particles is a source of entanglement in quantum mechanics. It is expressed in Bohm's formulation of the Einstein–Podolsky–Rosen problem [1]. A pair of spin-$\frac{1}{2}$ particles initially in

$$|\Psi_{\text{in}}\rangle = |k_1 k_2\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle), \qquad (1)$$

where 1 and 2 are the particle labels and k stands for the particle spatial mode, is split apart [2]. Delimited by Bell's inequality [3], a set of spin measurements along specific directions leads to a conflict between the predictions of quantum mechanics and the concepts of reality and locality as originally defined in Ref. [1]. Yet, experimental imperfection tends to erase the contradiction, and auxiliary assumptions are required to observe such conflicts. These assumptions generally lead to the communication loophole [4] and the detection loophole [5]. Polarization-entangled photons produced by parametric down-conversion have confirmed the predictions of quantum mechanics [6] and have even allowed the closure of the communication loophole [7]. The elusiveness of the detection loophole resides in the lack of photon counters which are both fast and efficient. To circumvent this problem, massive particles represent a promising alternative. The conservation of the particle number is a major key to close the detection loophole. In mesoscopic two-dimensional systems, this takes the form

* Corresponding author. Tel.: +1-650-723-3929; fax: +1-650-723-5320.

E-mail address: maitre@loki.stanford.edu (X. Maître)

1386-9477/00/$ - see front matter
PII: S1386-9477(99)00159-9

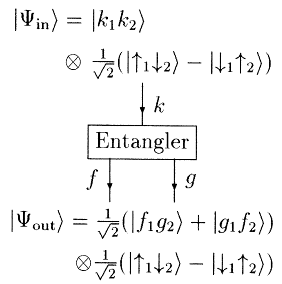

Fig. 1. Generation of entangled electrons in $|\Psi_{out}\rangle = |\Psi^+_{fg}\rangle$.

of conductance quantization. An electron which is injected through an input port must exit through an output port with unity quantum efficiency.

In a two-dimensional electron gas, the confinement in an edge channel, at a quantum point contact, or in a quantum dot can produce entanglement. When two electrons are confined so that they occupy the same state with the exception of the spin, they are described by the singlet spin state $|\Psi_{in}\rangle$. To implement Bohm's picture, one has to get them to fly apart and prepare the state $|\Psi^+_{fg}\rangle$:

$$|\Psi^+_{fg}\rangle = \tfrac{1}{2}(|f_1 g_2\rangle + |g_1 f_2\rangle) \otimes (|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle). \quad (2)$$

In the following, we assume that electrons are generated in this state $|\Psi^+_{fg}\rangle$ by an entangler in a 2DEG system (Fig. 1), where scattering can be neglected so two quiet streams, f and g, of distant spin and position-entangled electrons are produced.

2. Two-electron interferometry

The most straightforward way to test entanglement at the output of the above entangler is to collide the two electrons at a subsequent beam-splitter. When two electrons of the same spin collide and scatter, a destructive interference between probability amplitudes suppresses the probability that both particles are detected in the same direction [8]. However, when the two incoming electrons are position and spin-entangled, the former intuition cannot be carried on anymore. The electrons exit the beam-splitter through the same arm f, g:

$$|\Phi^+_{fg}\rangle = \tfrac{1}{2}(|f_1 f_2\rangle + |g_1 g_2\rangle) \otimes (|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle), \quad (3)$$

where the coefficients of the beam-splitter r and t are taken with the same half square modulus and overall phase factors are omitted. Such a collision exhibits the specific feature of bunching which characterizes the two-particle input state. The two output currents in modes f and g exhibit full shot noise (Fano factors $F_f = F_g = \langle \delta n_g^2\rangle / \langle n_g\rangle = 1$). This bosonic behaviour is explained by the symmetric orbital wave function which is necessarily associated with the antisymmetric spin singlet state of the electrons so that $|\Psi^+_{fg}\rangle$ describes a physical fermion state [9,10]. Following this guideline, electrons prepared in one of the symmetric triplet spin states are expected to antibunch since the associated orbital wave function is antisymmetric. The shot noise is consequently suppressed in the output currents ($F_f = F_g = 0$).

The statement based on the symmetry of the orbital wave function fails when one cannot guarantee one electron in each arm. When the above collision is reversed: $|\Phi^+_{fg}\rangle \to |\Psi^+_{fg}\rangle$, electrons prepared in a symmetric orbital wave-function antibunch. On the other hand, electrons prepared in a symmetric orbital wave function given by the state

$$|\Phi^-_{fg}\rangle = \tfrac{1}{2}(|f_1 f_2\rangle - |g_1 g_2\rangle) \otimes (|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle) \quad (4)$$

bunch when colliding. Therefore, one measurement after collision at a beam-splitter is not sufficient to distinguish between the singlet-spin states and the triplets. To describe the whole situation, we introduce the Bell-state notation both for spin and for position. We label the six possible states by their capital triplet contribution:

$$\begin{array}{ll} \text{Antisymmetric spin singlet} & \left\{\begin{array}{l} |\Phi^+_{fg}\rangle = |\phi^+_{fg}\rangle \otimes |\psi^-_{\uparrow\downarrow}\rangle, \\ |\Phi^-_{fg}\rangle = |\phi^-_{fg}\rangle \otimes |\psi^-_{\uparrow\downarrow}\rangle, \\ |\Psi^+_{fg}\rangle = |\psi^+_{fg}\rangle \otimes |\psi^-_{\uparrow\downarrow}\rangle, \end{array}\right. \\ \text{Symmetric orbital triplet} & \\ \text{Symmetric spin triplet} & \left\{\begin{array}{l} |\Phi^+_{\uparrow\downarrow}\rangle = |\psi^-_{fg}\rangle \otimes |\phi^+_{\uparrow\downarrow}\rangle, \\ |\Phi^-_{\uparrow\downarrow}\rangle = |\psi^-_{fg}\rangle \otimes |\phi^-_{\uparrow\downarrow}\rangle, \\ |\Psi^+_{\uparrow\downarrow}\rangle = |\psi^-_{fg}\rangle \otimes |\psi^+_{\uparrow\downarrow}\rangle, \end{array}\right. \\ \text{Antisymmetric orbital singlet} & \end{array} \quad (5)$$

where $|\phi^\pm_{fg}\rangle = \frac{1}{\sqrt{2}}(|f_1 f_2\rangle \pm |g_1 g_2\rangle)$, $|\psi^\pm_{fg}\rangle = \frac{1}{\sqrt{2}}(|f_1 g_2\rangle \pm |g_1 f_2\rangle)$, $|\psi^\pm_{\uparrow\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle \pm |\downarrow_1\uparrow_2\rangle)$, and $|\phi^\pm_{\uparrow\downarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\uparrow_2\rangle \pm |\downarrow_1\downarrow_2\rangle)$ are the four Bell states for spin and position. They form the complete maximally entangled basis of the two-electron Hilbert space associated, respectively, with position and with spin.

These states can be prepared from the entangled electrons generated at the entangler in the state $|\Psi^+_{fg}\rangle$, provided that we have a method to flip the spin of

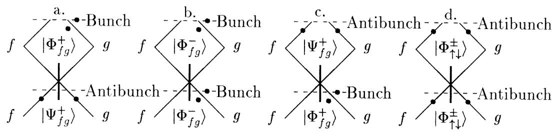

Fig. 2. Differentiation schemes of the six Bell-product states (a.) $|\Phi^+_{fg}\rangle$, (b.) $|\Phi^-_{fg}\rangle$ (c.) $|\Psi^+_{fg}\rangle$, (d.) $|\Phi^\pm_{\uparrow\downarrow}\rangle$ (or $|\Psi^+_{\uparrow\downarrow}\rangle$) by two series of measurements. Bunching behaviour is pictured by two dots in one arm, and antibunching by one dot in each arm.

either electron. We have just seen that colliding two electrons in $|\Psi^+_{fg}\rangle$ at a standard 50/50 beam-splitter prepares the state $|\Phi^+_{fg}\rangle$. If the spins in the arm f are flipped, the electrons are prepared in the state $|\Phi^+_{\uparrow\downarrow}\rangle$, and they antibunch at the beam-splitter into the same state $|\Phi^+_{\uparrow\downarrow}\rangle$. If they are flipped after the collision that leads to $|\Phi^+_{fg}\rangle$, then $|\Phi^-_{fg}\rangle$ is generated and the electrons bunch at a subsequent beam-splitter into the same state $|\Phi^-_{fg}\rangle$. To generate the last two triplet spin states, a polarizing beam-splitter is required to spatially separate the two spin components in f into a spin-up path and a spin-down path. Then, only the spin-down component in f can be selectively flipped. Lastly, the spin-up and spin-down paths are recombined. If the input state is $|\Psi^+_{fg}\rangle$, then the prepared state following this spin-dependent flip is $|\Psi^+_{\uparrow\downarrow}\rangle$. If the input state is $|\Phi^+_{\uparrow\downarrow}\rangle$, then the prepared state is $|\Phi^-_{\uparrow\downarrow}\rangle$. A polarizing beam-splitter could be realized by the adjunction of a micromagnet designed on the top of the two-dimensional electron gas and close to a pair of split gates. With ferromagnetic materials like dysprosium, it is possible to locally enhance a small external magnetic field by a few Tesla [11,12]. Since the spin degeneracy is locally lifted, the transmission and reflection coefficients of the corresponding beam-splitter are spin-dependent.

The three orbital triplet states $|\Phi^\pm_{fg}\rangle$ and $|\Psi^+_{fg}\rangle$ can be distinctively detected from the three spin triplet states $|\Phi^\pm_{\uparrow\downarrow}\rangle$ and $|\Psi^+_{\uparrow\downarrow}\rangle$ with two sets of measurements. Before the beam-splitter, noise measurements differentiate antibunched electrons in the states $|\Phi^\pm_{\uparrow\downarrow}\rangle$, $|\Psi^+_{\uparrow\downarrow}\rangle$, or $|\Psi^+_{fg}\rangle$ from bunched electrons in the states $|\Phi^\pm_{fg}\rangle$. After the beam-splitter, subsequent noise measurements differentiate the two remaining possibilities. The four possible configurations are summarized in Fig. 2.

The spin triplet states $|\Phi^\pm_{\uparrow\downarrow}\rangle$ and $|\Psi^+_{\uparrow\downarrow}\rangle$ can then be discriminated by first applying the inverse transformation required for their preparation and then by detecting bunching or antibunching before and after the beam-splitter. The correct set of operations followed by two series of noise measurements up and down stream from the beam-splitter allows one to distinguish without any ambiguity the type of entanglement between the two electrons.

3. A new test of Bell's inequality

The most striking evidence of entanglement remains the violation of Bell's inequality. It demonstrates the non-locality of quantum theory. The anticorrelations of the electron spins of each entangled pair should be measured in different bases at the outputs of the entangler while the electrons fly apart. We must extract from the data a signal which is limited by any local theories, but which takes greater values if the two electrons are quantum mechanically entangled. This is the essence of Bell's inequality.

As in optics, the setup requires a polarizing beam-splitter in both arms f and g that directs the spin components towards two different output ports, $f_\downarrow$, $f_\uparrow$ and $g_\downarrow$, $g_\uparrow$. Spin-down electrons are reflected into $f_\downarrow$ or $g_\downarrow$ while spin-up electrons are transmitted into the two other ports $f_\uparrow$ and $g_\uparrow$. Noise correlation measurements demonstrate the anticorrelation of the two electron spins: $\langle \delta n_{f_\downarrow} \delta n_{g_\uparrow} \rangle = \langle \delta n_{f_\uparrow} \delta n_{g_\downarrow} \rangle = +1/4$ whereas $\langle \delta n_{f_\downarrow} \delta n_{g_\downarrow} \rangle = \langle \delta n_{f_\uparrow} \delta n_{g_\uparrow} \rangle = -1/4$.

In 2DEG systems, it is a priori not easy to independently rotate the spins of electrons traveling through modes f and g as half wave plates do for photons. The quantization axis defined by a small external magnetic field cannot be locally tuned. However, when the spin degeneracy is transiently lifted, free evolution of the two spin components results in a rotation in the plane perpendicular to the magnetic field $\boldsymbol{B}$. This transformation can be induced independently on each arm with two additional micromagnets (Fig. 3). The induced rotation angles φ_f and φ_g can be tuned by applying a bias voltage on two gates in order to vary the path length taken by the electrons under the micromagnets. These rotations are equivalent to tuning the wave plate thicknesses. To read out such transformations in optics, the polarizing beam-splitters must

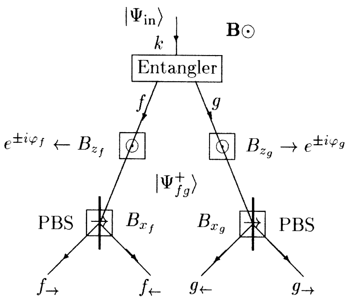

Fig. 3. Experimental scheme for a test of Bell's inequality in a two-dimensional electron gas. The two spin components are phase shifted by $\pm\varphi_f$ and $\pm\varphi_g$ in a locally enhanced magnetic field in each path, $B_{\perp_f}$ and $B_{\perp_g}$, generated by two micromagnets. The spin components are split in a perpendicular magnetic field, $B_{\|_f}$ and $B_{\|_g}$, locally generated by two other micromagnets. The spin anticorrelations are read out through noise correlation measurements.

be turned by 45°. In mesoscopic physics, the local magnetic field for the polarizing beam-splitters must be generated by a second type of micromagnets in the plane of the 2DEG along the x direction. Therefore, if the global magnetic field in the z direction is negligible compared to the field in the x direction, then the spin is measured along the new local eigenvector directions $\{|\rightarrow\rangle, |\leftarrow\rangle\} = \{(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}, (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}\}$.

We now examine noise correlations between different ports which can lead to a conflict with locality. The singlet state is rotationally invariant, and it can be rewritten equivalently in the $\{|\rightarrow\rangle, |\leftarrow\rangle\}$ basis as $|\psi^-_{\rightarrow\leftarrow}\rangle$. By rotating the spins we can register the noise correlations as a function of the phase shifts φ_f and φ_g experienced by the electrons:

$$\langle \delta n_{f_\leftarrow} \delta n_{g_\rightarrow} \rangle = \langle \delta n_{f_\rightarrow} \delta n_{g_\leftarrow} \rangle = +\tfrac{1}{4}\cos(\varphi_f - \varphi_g), \quad (6)$$

$$\langle \delta n_{f_\leftarrow} \delta n_{g_\leftarrow} \rangle = \langle \delta n_{f_\rightarrow} \delta n_{g_\rightarrow} \rangle = -\tfrac{1}{4}\cos(\varphi_f - \varphi_g). \quad (7)$$

Following the generalization of Clauser et al. [13], we define equivalently for the spins of the electron pairs the correlation function:

$$E(\varphi_f, \varphi_g) = \langle \delta n_{f_\rightarrow} \delta n_{g_\rightarrow} \rangle + \langle \delta n_{f_\leftarrow} \delta n_{g_\leftarrow} \rangle - \langle \delta n_{f_\rightarrow} \delta n_{g_\leftarrow} \rangle - \langle \delta n_{f_\leftarrow} \delta n_{g_\rightarrow} \rangle. \quad (8)$$

It is a function of the measurement directions given by the angles φ_f and φ_g for each electron. Any local theories must then satisfy the inequality

$$-2 \leqslant S(\varphi_f, \varphi_g, \varphi'_f, \varphi'_g) \leqslant +2, \quad (9)$$

with $S(\varphi_f, \varphi_g, \varphi'_f, \varphi'_g) = E(\varphi_f, \varphi_g) - E(\varphi_f, \varphi'_g) + E(\varphi'_f, \varphi_g) + E(\varphi'_f, \varphi'_g)$, where the correlation function is measured for different angles, φ_f, φ_g, φ'_f, and φ'_g.

For the set of angles $\varphi_f - \varphi_g = \varphi'_f - \varphi_g = \varphi'_g - \varphi'_f$:

$$S(\varphi_f, \varphi_g) = \cos 3(\varphi_f - \varphi_g) - 3\cos(\varphi_f - \varphi_g). \quad (10)$$

A maximal violation of Bell's inequality can be reached for $\{\varphi_f, \varphi_g\} = \{\pi/8, 3\pi/8\}$, $\{3\pi/8, \pi/8\}$, $S(\varphi_f, \varphi_g) = \pm 2\sqrt{2}$.

This proposed test of Bell's inequality is very challenging. Electron beam-splitters have been successfully tested [14,15] but vertical and horizontal electron polarizing beam-splitters must still be demonstrated. Nevertheless, they rely on the same mechanism of reflection and transmission at a potential barrier. The rotation induced by the free evolution of the spins is normally free of imperfection as long as the electrons follow identical paths and experience the same magnetic fields.

In summary, the electron entanglement in 2DEG systems can be demonstrated by noise measurement techniques. It exploits the unique properties of Fermi-degenerate electrons which allows the possibility of a regulated source of entangled particles. This is radically different from previous experiments with massive particles [16–19] or photons [6,7,20,21]. Statistics of single-particle events are not required. Events are continuously registered and entanglement is verified by noise correlation measurements. This makes possible a new test of Bell's inequality with the closure of the detection loophole.

Acknowledgements

We would like to thank Edo Waks for helpful comments.

References

[1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47 (1935) 777.
[2] D.J. Bohm, Quantum Theory, Constable, London, 1954.
[3] J.S. Bell, Physics 1 (3) (1964) 195.
[4] P.G. Kwiat, P.H. Eberhard, A.M. Steinberg, Raymond Y. Chiao, Phys. Rev. A 49 (5) (1993) 3209.

[5] P.M. Pearle, Phys. Rev. D 2 (8) (1970) 1418.
[6] P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, Y. Shih, Phys. Rev. Lett. 75 (24) (1995) 4337.
[7] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. 81 (23) (1998) 5039.
[8] R.C. Liu, B. Odom, Y. Yamamoto, S. Tarucha, Nature 391 (1998) 263.
[9] M. Michler, K. Mattle, H. Weinfurter, A. Zeilinger, Phys. Rev. A 53 (3) (1996) R1209.
[10] G. Burkard, D. Loss, E.V. Sukhorukov, Noise of Entangled Electrons: Bunching and Antibunching. cond-mat/9906071, June 1999.
[11] P.D. Ye, D. Weiss, R.R. Gerhardts, M. Seeger, K. von Klitzing, K. Eberl, H. Nickel, Phys. Rev. Lett. 74 (15) (1995) 3013.
[12] V. Stepankin, Physica B 211 (1995) 345.
[13] J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. 23 (15) (1969) 880.
[14] M. Henny, S. Oberholzer, C. Strunk, T. Heinzel, K. Ensslin, M. Holland, C. Schönenberger, Science 284 (5412) (1999) 296.
[15] W.D. Oliver, J. Kim, R.C. Liu, Y. Yamamoto, Science 284 (5412) (1999) 299.
[16] L.R. Kasday, J.D. Ullman, C.S. Wu, Nuovo. Cimento Soc. Ital. Fis. 25B (2) (1975) 633.
[17] M. Lamehi-Rachti, W. Mittig, Phys. Rev. D 14 (10) (1976) 2543.
[18] E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond, S. Haroche, Phys. Rev. Lett. 79 (1) (1997) 1.
[19] Q.A. Turchette, C.S. Wood, B.E. King, C.J. Myatt, D. Leibfried, W.M. Itano, C. Monroe, D.J. Wineland, Phys. Rev. Lett. 81 (17) (1998) 3631.
[20] S.J. Freedman, J.F. Clauser, Phys. Rev. Lett. 28 (14) (1972) 938.
[21] A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. 47 (7) (1981) 460.