On the Classical Capacity of a Quantum Multiple-Access Channel

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Abstract — **We analyze quantum adder channel, i.e. the extention of a classical multiple-access additive binary channel to the quantum regime, in which quantum bits (qubits) rather than classical bits are trasmitted. Quantum entanglement between different trasmitters and/or the receiver results in a significant expansion of the capacity region of the channel, in comparison with its classical counterpart, a property which can be used, e.g., to considerably improve bandwidth efficiency of fiber communication networks in the future.**

I. Summary

The capacity region of a general multiple-access channel was determined in [1] for pure signal states and product state encoding, i.e. when Holevo bound [2] holds.

The quantum adder multiple-access channel (QMAC) is derived from the classical additive noiseless channel with binary user's inputs $X_1,...X_L$ and the receiver output

$$Y = X_1 + ... + X_L \,, \tag{1}$$

which is equivalent to the channel randomly permuting the inputs. The capacity region is well known for the classical channel with two users and is plotted in Fig. 1 (dotted line), the maximum rate-sum being 3/2.

In the quantum case, each input X_i is a qubit rather than a classical bit, i.e. a quantum system with a two-dimensional Hilbert state. Either Boolean value corresponds to one of two mutually orthogonal states of the system, such as a clockwise and counterclockwise polarized photon.

To generalize channel additivity to the quantum case, we assume that the channel randomly permutes the user's qubits.

To encode infomation, uncoordinated users are assumed to apply data-dependent local unitary operators to their qubits, so that the encoded state (before random perbutation of qubits) is obtained by applying the product of these local operators to the initial quantum state. We consider the following three degrees of entanglement for the initial state of quantum adder channel: without either user-user or user-receiver entanglement, with user-user entanglement only, and with both user-user and user-receiver entanglement.

With no entanglement whatsoever, the capacity region is the same as for the classical adder channel, which also provides intuitive support for our generalization of the classical adder channel to the quantum case.

With user-user entanglement only, the capacity region

$$R_1 \leq 2 \,, \quad R_2 \leq 2 \,, \quad R_1 + R_2 \leq 2 \tag{2}$$

is achieved, e.g. by the encoding technique similar to the superdense quantum coding [3].

With single-bit user-receiver entanglement, the capacity region is given by

$$R_1 \leq 2 \,, \quad R_2 \leq 2 \,, \quad R_1 + R_2 \leq 5/2 \,. \tag{3}$$

In general for any number of users on the quantum adder channel with one-bit user-receiver entanglement, users achieve at least one extra bit of rate-sum (shared in arbitrary proportion among them) in comparison with the classical adder channel.

With full (two-bit) user-receiver entanglement, the capacity region is defined by

$$R_1 \leq 2 \,, \quad R_2 \leq 2 \,, \quad R_1 + R_2 \leq 4 - H\,(3/4) \approx 3.19 \tag{4}$$

References

[1] A. Winter, "The Capacity of the Quantum Multiple Access Channel", quant-ph/9807019.

[2] A. S. Holevo, "The Capacity of the Quantum Channel with General Signal States", IEEE Trans. Inf. Theory, vol. 44, no. 1, pp. 269–273, 1998.

[3] C. H. Bennett and S. J. Wiesner, "Communication via one- and two-particle operators on EPR states", Phys. Rev. Lett., vol 69, pp. 2881-4, 1992.

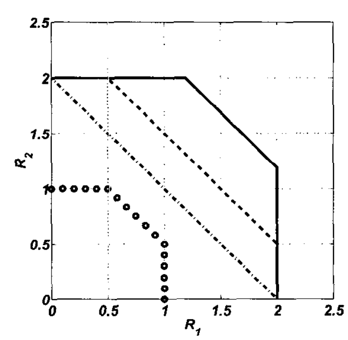

Figure 1: The capacity region for the classical as well as quantum adder channel without entanglement (dotted line), and for the quantum adder channel with the following user-receiver entanglement: zero bits, i.e. user-user entanglement only (dash-dotted line), one-bit (dashed line), and full (two-bit) entanglement (solid line).

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