**Ezaki *et al.* Reply:** In our recent Letter [1], we proposed generation of a phase state by two-photon absorption. Alexanian and Bose [2] called attention to the fact that the steady state solution of the master equation does not satisfy purity condition $\mathrm{Tr}\hat{\rho}^2 = 1$. We thank them for pointing out this issue. The purity and orthogonality of the output states were not discussed in [1]. Here we will supplement this issue by answering the Comment [2].

Our analytical results [Eq. (4)] in [1] are rewritten as

$$\langle 0|\rho(\infty)|1\rangle = \langle 0|\rho(0)|1\rangle + \sum_{q=1}^{\infty}\prod_{r=1}^{q}\sqrt{1-\frac{1}{4r^2}}\,\langle 2q|\rho(0)|2q+1\rangle, \tag{1}$$

$$= e^{-|\alpha|^2-i\phi}|\alpha|\left\{1+\sum_{q=1}^{\infty}(|\alpha|^2)^{2q}\frac{\sqrt{3}}{2}\,\frac{\sqrt{15}}{4}\cdots\frac{\sqrt{(2q+1)(2q-1)}}{2q}\,\frac{1}{\sqrt{2q!}}\,\frac{1}{\sqrt{(2q+1)!}}\right\} \tag{2}$$

for coherent light input, where $\alpha = |\alpha|e^{i\phi}$. The terms in the curly brackets are easily rewritten by a modified Bessel function $I_0$ as $\langle 0|\rho(\infty)|1\rangle = e^{-|\alpha|^2-i\phi}|\alpha|I_0(|\alpha|^2)$, which agree with the results obtained by Simaan and Loudon [3] and by Alexanian and Bose [2]. Its asymptotic form for $|\alpha| \gg 1$ is reduced to $\langle 0|\rho(\infty)|1\rangle = e^{-i\phi}\sqrt{2\pi} \neq e^{-i\phi}/2$. Therefore, the output state for $|\alpha| \gg 1$ is not a pure state. However, for an input coherent state with $|\alpha| \sim 1$, a phase state is approximately realized by two-photon absorption with sufficiently small error rates, as shown below.

Figure 1 shows $\langle\phi|\rho|\phi\rangle$ (solid circles) for the output state, where $|\phi\rangle$ is a pure phase state $|\phi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $\rho$ is obtained from the analytical results for $\phi = 0$ in Eq. (2). It gives the measure of the overlap between the output quasiphase state and the pure phase state. The star in the figure shows the value for an intense coherent state, i.e., for $|\alpha| \gg 1$. The overlap takes a maximum $\langle\phi|\rho|\phi\rangle = 0.966$ at $\alpha = 1$ and decreases monotonically with increasing $|\alpha|$.

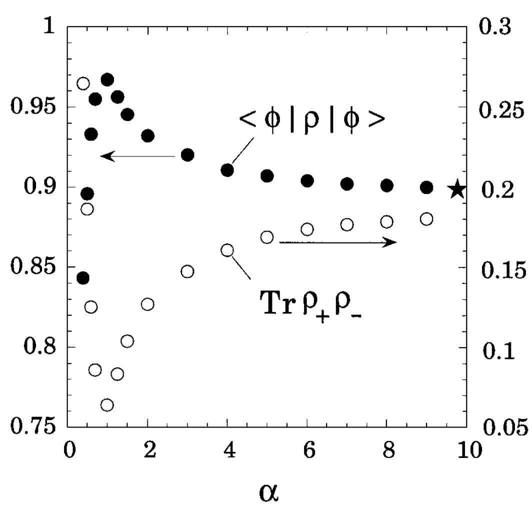


FIG. 1. $\langle\phi|\rho|\phi\rangle$ and $\mathrm{Tr}\rho_+\rho_-$ as a function of $\alpha$ for the steady state, where $\rho_+$ and $\rho_-$ are the output states corresponding to input coherent states with the initial phases $\phi = 0$ and $\pi$, respectively.

In view of practical application to the quantum cryptography, any real photon source does not produce an exactly pure single-photon number state nor phase state. Therefore, an important criterion for a new source is the degree of the orthogonality between two quasiphase states, which is also shown in Fig. 1. The overlap between the two states takes a minimum 0.064 at $\alpha = 1$ corresponding to the maximum of $\langle\phi|\rho|\phi\rangle$. When we use these two states as two orthogonal bases of quantum cryptography, the error radte is roughly estimated by $\mathrm{Tr}\rho_+\rho_-$. The value is not much worse than the quantum cryptography using a single-photon polarization state, if we take into account the polarization error or phase error encountered in a practical system.

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