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Violation of Bell's Inequality by a Generalized Einstein-Podolsky-Rosen State Using Homodyne Detection

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Using homodyning with weak coherent fields and photon counting, we have observed violations of Bell-type inequalities by the generalized Einstein-Podolsky-Rosen state produced in a pulsed nondegenerate optical parametric amplifier, as predicted by Grangier *et al.* [Phys. Rev. A **38**, 3132 (1988)]. The maximum observed visibility of the interference pattern was (89 ± 4)%. This interference can be regarded as a manifestation of nonlocality in the sense described by Banaszek and Wodkiewicz [Phys. Rev. A **58**, 4345 (1998)]. We have investigated the interference both theoretically and experimentally and have measured the influence of dispersion and phase matching.

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The venerable Einstein-Podolsky-Rosen (EPR) state was introduced more than sixty years ago [1] to point out the apparent incompleteness of the quantum mechanical description of nature. The EPR program was extended by Bell to demonstrate the incompatibility of quantum mechanics with local realistic desriptions by means of the celebrated Bell inequalities [2]. Nonetheless the state originally proposed by EPR was considered by Bell to be incapable of exhibiting contradiction between quantum mechanics and local realism [3]. Quantum optical demonstrations of Bell-type inequality violations, using two-photon states analogous to Bohm-type spin-entangled states, have provided strong evidence in favor of quantum mechanics [4,5].

By contrast, an experimental realization of an optical analog of the EPR state itself and of the ensuing EPR paradox was reported by Ou *et al.* [6]. More recently, such a state served as the basis of an unconditional quantum teleportation experiment [7]. Despite great advances in the understanding and experimental investigation of the EPR state, opinions still differ on whether such a state exibits nonlocality or not. On the one hand, since the Wigner function $W_{\mathrm{EPR}}(\alpha_1, \alpha_2)$ [6] of the (generalized) EPR state,

$$W_{\mathrm{EPR}}(\alpha_1, \alpha_2) = \frac{4}{\pi^2} \exp[-2\cosh 2r(|\alpha_1|^2 + |\alpha_2|^2) + 2\sinh 2r(\alpha_1\alpha_2 + \alpha_1^*\alpha_2^*)], \quad (1)$$

is everywhere non-negative, it has been argued [3,6,8] that a local realistic interpretation of phenomena associated with the EPR state should be possible (here r is the parametric gain). On the other hand, Grangier *et al.* [9] showed that, when homodyned with two weak coherent local oscillators (see Fig. 1), the EPR state exhibits a sinusoidal modulation of the photon counting rate as a function of the relative phases of the EPR and coherent states. They explicitly constructed a Bell-type inequality and showed that the violation of this inequality for the EPR state is the same as in Bohm-type experiments with two entangled spins [4].

More recently in a series of papers Banaszek and Wódkiewicz [10] argued that negativity of the Wigner function is not required for violation of the Bell-type inequalities. In fact, they showed that the photon detection probabilities can be directly related to certain phase space densities and that for the EPR state in particular a Bell-type inequality can be violated. They made use of the observation that the Wigner function can be expressed as an expectation value of the displaced parity operator. Since the parity operator is a dichotomic observable, they were able to construct a Bell-type inequality similar to the one for two entangled spin-1/2 particles. Although the Banaszek and Wódkiewicz [10] inequalities are different from that of Grangier *et al.* [9], they are again violated for the (generalized) EPR state in a homodyne detection scheme of the type shown in Fig. 1. In both cases the degree of violation of the local realistic bound is most pronounced for small values of parametric gain r.

Experiments [6,7] were based on homodyning the EPR state with strong local oscillator fields and subsequent detection with linear photodiodes, whereas Grangier *et al.* [9] and Banaszek and Wódkiewicz [10] require weak local oscillators and photon counting in order to discriminate between the quantum mechanical and local realistic descriptions of nature. In this Letter we report the first experimental demonstration of the violation of a Bell inequality for the EPR state as predicted in Refs. [9,10].

Consider the arrangement shown in Fig. 1: a nonlinear crystal functioning as a parametric down-converter (PDC) is pumped by a short coherent pulse of light that generates two correlated light beams, a signal, and an idler. Each of these beams is mixed with a weak coherent pulse on a 50:50 beam splitter (BS) and one of the outputs of each beam splitter is measured with a photon counting detector.

There are certain advantages to using short pulses to pump the PDC and for the local oscillators. In order to obtain high visibility of interference it is vital to properly match the local oscillator modes and the modes of the down-converted photons in both space and time; it is difficult to do this with a narrow band cw laser given the long dead time of single-photon detectors and the need to maintain an adequate count rate in the experiment. The original proposal [9] treated cw fields and therefore the analysis is not directly applicable to the experiment with ultrashort ($\sim$100 fs) pulses that we report. We deal with the total number of detected coincidences instead of the time-correlation function and we include the effects of dispersion, and this distinguishes the pulsed light experiments from the cw treatment [9].

Theoretical modeling of our experiment follows the approach of Refs. [11–13]. Let us suppose that idler and signal fields produced in spontaneous down-conversion are mixed with two coherent pulses on 50:50 beam splitters and a photon counting detector is located at the output of each beam splitter (see Fig. 1). According to Ref. [9], the observable corresponds to a coincident count between the detectors D_1 and D_2 in our experiment. Banaszek and Wódkiewicz [10] demonstrated that measurement of the expectation value of the displaced parity operator $\hat{D}(\alpha_1)(-1)^{\hat{n}_1}\hat{D}^\dagger(\alpha_1)\hat{D}(\alpha_2)(-1)^{\hat{n}_2}\hat{D}^\dagger(\alpha_2)$ in the arrangement of Fig. 1 gives the scaled Wigner function $\frac{\pi^2}{4}W_{\mathrm{EPR}}(\alpha_1,\alpha_2)$ of the EPR state, while measurement of the expectation value of the displaced zero-count operator $\hat{D}(\alpha_1)|0\rangle_1\langle 0|_1\hat{D}^\dagger(\alpha_1)\hat{D}(\alpha_2)|0\rangle_2\langle 0|_2\hat{D}^\dagger(\alpha_2)$ gives the Q function. It is easy to see that the measurement of the probability $P(\alpha_1,\alpha_2)$ of detecting a coincident event between the detectors D_1 and D_2 is equivalent to measuring the difference between the Q and the scaled Wigner functions: $P(\alpha_1,\alpha_2) = Q_{\mathrm{EPR}}(\alpha_1,\alpha_2) - \frac{\pi^2}{4}W_{\mathrm{EPR}}(\alpha_1,\alpha_2)$ if one keeps in mind that the r^2, $|\alpha_1|^2$, and $|\alpha_2|^2$ are all much smaller than unity. After a straightforward but somewhat lengthy derivation we find that $P(\alpha_1,\alpha_2)$ is a sinusoidal function of the phase difference θ between the down-converted light and the coherent pulse:

$$P(\alpha_1,\alpha_2) = M[1 + V\cos(\phi_1 + \phi_2 - \theta)], \quad (2)$$

where $\phi_j \equiv \arg(\alpha_j)$, $j = 1,2$, θ is the common phase of the EPR state, and M is a constant. The visibility V of the interference depends on several parameters, such as the length and dispersions of the crystal, pump, filters, etc. [14]. It is maximum when the coincidence rate due to coherent light only is equal to the coincidence rate due to the EPR state. The visibility goes to zero as the spatiotemporal overlap between the down-converted photon wave packets and the local oscillator pulses decreases.

Grangier *et al.* [9] have constructed the following Bell-type inequality:

$$-2 \le S \le 2\,, \quad (3)$$
$$S \equiv E(\phi_1,\phi_2) - E(\phi_1,\phi_2') + E(\phi_1',\phi_2) + E(\phi_1',\phi_2')\,,$$

where $E(\phi_1,\phi_2)$ depends on $P(\alpha_1,\alpha_2)$ for different sets of detectors and in our case is given by $E(\phi_1,\phi_2) = V\cos(\phi_1 + \phi_2 - \theta)$. The violation of the inequality (3) is maximum for the following phase angles:

$$\phi_2 = \theta - \phi_1 + \pi/4\,,$$
$$\phi_1' = \phi_1 - \pi/2\,, \quad (4)$$
$$\phi_2' = \phi_1 - \pi/2\,,$$

with $S = V2\sqrt{2}$. Therefore, whenever $V > 1/\sqrt{2}$, $S > 2$ and the inequality (3) is violated [9,13] (a discussion of supplementary assumptions relevant to the Bell-type inequalities in experiments using homodyning techniques is given in Refs. [9,15,16]).

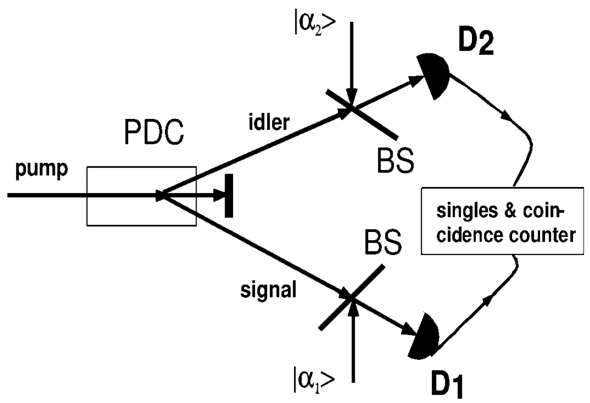

FIG. 1. Schematic of the scheme to test Bell-type inequality for the EPR state. Photon pairs from a parametric down-converter (PDC) are mixed with weak coherent-state pulses at beam splitters (BS), and the numbers of photon counts are recorded at detectors D_1 and D_2.

The experimental setup is shown in Fig. 2. The output power of the Ti:sapphire laser (Ti:Spp), consisting of a mode-locked train of pulses centered at 810 nm with bandwidth of 10 nm, is about 1.8 W on average. The light is used to pump a 0.7 mm BBO crystal [second-harmonic generation (SHG)] (cut and aligned for collinear type-I phase matching) which generates a second harmonic of average wavelength 405 nm with about 15% efficiency. The second-harmonic crystal is in effect the "input beam splitter" of our two-color interferometer. After the crystal, both fundamental and second harmonic copropagate and are subsequently separated by a dichroic beam splitter (DBS). After the red light is blocked by a blue color glass filter (BG), the 405 nm light is focused onto the parametric down-converter (PDC), consisting of a 2.5 mm thick (BBO) crystal cut and aligned for type-II phase matching. In order to efficiently couple the down-converted light into a single mode fiber (see below), we focused the pump light, so that the beam had a Rayleigh range approximately equal to the crystal length.

The orthogonally polarized, collinear signal and idler beams produced in the crystal are passed through an 8 mm thick quartz plate (QP), aligned so as to compensate for the birefringence of the BBO crystal. The beams are then recombined on a 50:50 beam splitter (BS) with the red light that has been highly attenuated and passed through a calibrated controllable delay line (CDL) consisting of a mirror on a translation stage and a piezoelectric transducer (PZT). Half-wave and quarter-wave plates allow us to change the relative phase $\phi_1 - \phi_2$ between the two coherent pulses. One of the two beam-splitter outputs is passed through dichroic mirrors that reflect the 405 nm light and is directed onto a polarizing Glan-Thompson beam splitter (PBS). The outputs of the PBS are focused onto the two avalanche photodiodes D_1 and D_2 acting as photon counting detectors. Broad-band interference filters (BF) of 40 nm bandwidth are placed in front of the photodiodes, so as to reduce contributions from the scattered 405 nm radiation. Before the PBS, narrow-band filters (NF) of different bandwidths may be inserted (see below). Photon detection events in the photon counting detectors produce output pulses that are passed through discriminators emitting 2 ns long NIM pulses. The outputs of the discriminators are fed into a coincidence "AND" gate, and coincident and single counts are registered [17].

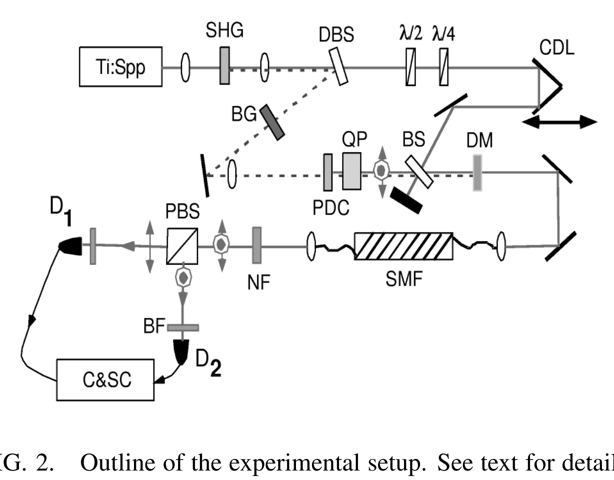

FIG. 2. Outline of the experimental setup. See text for details.

The efficiency of a homodyne detector depends on the degree to which signal and local oscillator modes are matched. In order to achieve a good match, we spatially and spectrally filtered both modes using single mode fibers and interference filters [17]. This avoids the problem that the spatial mode of the signal and idler photons from the down-converter do not have a Gaussian profile [18]. In fact, complete mode matching requires in addition the use of a dispersive element to adjust the relative spectral phase of the modes. For this purpose we used a piece of quartz of a dispersive element (a quartz plate, QP in Fig. 2) to adjust the relative spectral phases of the modes. With attention paid to mode matching, we were able to significantly increase the visibility of the interference pattern in our experiment. Moreover, with single spatial mode filtering, the measured visibilities were usually close to the calculated ones, although they were below unity due to uncompensated dispersion in the down-converter.

In order to check the effects of imperfect mode matching, we have recorded interference patterns as functions of $\phi_1 + \phi_2$ for four values of the narrow-band filter bandwidth: 3.5, 6, 16, and 40 nm. Plot (a) of Fig. 3 shows the recorded interference pattern with the 3.5 nm filter at the maximum of the visibility curve according to plot (b). After flipping the half-wave plate and the quarter-wave plate we shifted $\phi_1 - \phi_2$ by π and $\pi/2$, respectively, and observed similar interference curves. In plot (a) the sinusoidal curve is the best (least squares) fit when visibility, amplitude, and phase are treated as adjustable parameters. In plot (b) the theoretical curve is based on the expression that we derived with no adjustable parameters. The visibility V reached 89($\pm$4)%, so that $S_{\mathrm{exp}} = 2.46 \pm 0.06$, in violation of the Bell inequality (3). As the bandwidth of the filter increased, the visibility of the interference dropped markedly as shown by the experimental points in Fig. 4. The calculations we performed (solid line in Fig. 4) show that both the walk-off between the pump pulse and the signal and idler wave packets, resulting from the dispersion of the crystal, contribute to the reduction of visibility. On the basis of theoretical calculations, we attribute the fact that the observed visibility of interference with the 40 nm filter is significantly lower than the theoretical value to the combination of some unknown birefringence in the optical elements, to possible imperfect cancellation of signal-idler walk-off by the quartz plate, and to deviation of the filter transmission function from a true Gaussian. Despite this, the trend of increasing observed visibility with smaller filter bandwidth is apparent. Single-photon counts in all four channels did not show any modulation at all as the function of ϕ_1, ϕ_2, which lends support to the "no-enhancement" assumption of Grangier *et al.* [9].

In conclusion, we have for the first time observed the violation of a Bell-type inequality when both signal and

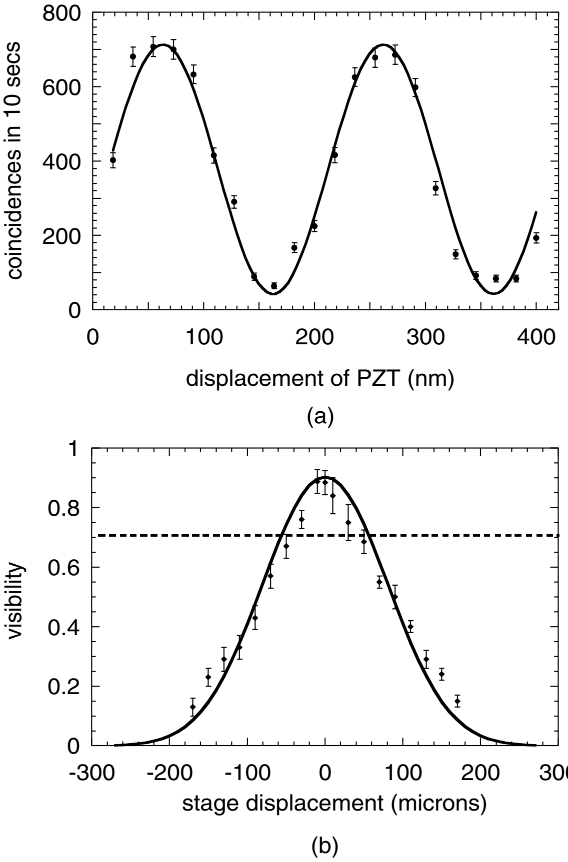

FIG. 3. (a) Interference pattern observed as the function of the displacement of a PZT-mounted mirror with 3.5 nm filter in place (circles). The sinusoidal curve is the least squares fit based on expression (2), when visibility, amplitude, and phase are treated as adjustable parameters, and is used to determine the observed visibility. (b) Diamonds correspond to the observed values of the visibility of the interference pattern as a function of the relative delay between the local oscillators and the down-converted photons. The solid curve is our theoretical calculation with no adjustable parameters. The dashed line corresponds to the highest visibility consistent with local realistic theories [9,13].

idler beams produced by a parametric down-converter were registered by means of photon counting homodyne detection. The interference between radiation produced by spontaneous down-conversion and weak coherent light can give rise to a manifestation of locality violations via the Bell inequalities, as was pointed out by Grangier *et al.* [9]. Moreover, such a violation can be interpreted in terms of positive-definite phase space distributions as pointed out by Banaszek and Wódkiewicz [10].

The question remains as to what degree our experiment demonstrates a violation of local realism. Certainly there exists a similar loophole to many prior experiments based on photon pairs generated via spontaneous down-conversion: we did not vary the detector parameters (in our case the phase settings of the local oscillators) independently and rapidly enough to ensure spacelike separation of the individual homodyne apparatuses. Nonetheless, there

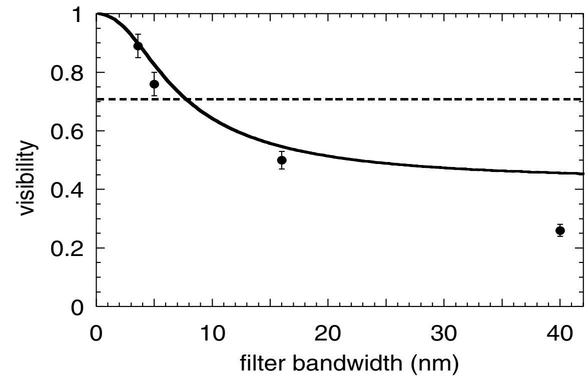

FIG. 4. Circles correspond to observed visibilities of the interference at their maxima for different values of the filter bandwidth. The solid line is obtained from our theoretical calculations and the dashed line has the same significance as in Fig. 3.

is no identifiable physical mechanism of sufficient strength that could cause the independent polarization channels to somehow learn of the phase setting of the other. It requires a supplementary assumption that no hidden communication mechanism exists before we may unequivocally claim to have demonstrated a contradiction with locality. Despite this, our experiments clearly indicate that such a novel test of local hidden variables is possible.

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