

Violation of a Bell-type inequality in the homodyne measurement of light in an Einstein-Podolsky-Rosen state

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An optical Einstein-Podolsky-Rosen (EPR) state is produced in the spontaneous parametric down-conversion of an ultrashort classical pump pulse. It is shown both theoretically and experimentally that this results in the violation of a Bell-type inequality of the kind proposed by Grangier *et al.* The experiment is based on measuring interference between the light in the EPR state and weak light pulses in a coherent state. The maximum observed visibility of the interference pattern was $(89 \pm 4)\%$. This interference can be regarded as a manifestation of nonlocality in the sense described by Banaszek and Wodkiewicz, even though the EPR state has a positive Wigner function. We develop the theory of two-channel homodyne detection with ultrafast pulses and then apply it to the EPR state generated in a type-II collinear geometry. Particular attention is paid to the investigation of how dispersion and phase matching influence the violation of local realism.

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I. INTRODUCTION

In 1935 Einstein, Podolsky, and Rosen (EPR) [1] pointed out what they considered to be an incompleteness of the quantum mechanical description of nature. They considered a state of two particles entangled in their positions and momenta with a wave function of the kind $\sim \delta(x_1 - x_2 - L)$. Because of the δ function, a measurement of the position of one particle provides an exact value of the position of the other one, and the same is true for measurements of the momenta of these two particles. EPR suggested simultaneously measuring the momentum of one of the particles and the position of the other. In this way, they argued, one can infer both the position and the momentum of one particle, even though operators \hat{x} and \hat{p} do not commute. EPR concluded from this example that “*the quantum mechanical description of physical reality given by the wave function is not complete*” [1].

The EPR program was later extended by Bell [2], who demonstrated the incompatibility of quantum mechanics with local realistic descriptions of nature by means of the celebrated Bell inequalities. Nonetheless, the state originally proposed by EPR was considered by Bell to be incapable of exhibiting the contradiction between quantum mechanics and local realism [3]. Several quantum optical demonstrations of the violations of Bell-type inequalities provide strong evidence in favor of quantum mechanics [4,5]. The two-photon states used in these experiments are analogous to Bohm-type spin-entangled states.

By contrast, an experimental realization of the EPR state itself and of the ensuing EPR paradox was reported by Ou *et al.* [6]. More recently, such a state served as the basis of an unconditional quantum teleportation experiment [7]. Despite great advances in the understanding and the experimental investigation of the EPR state, opinions still differ on whether such a state exhibits nonlocality or not. On the one hand, since the Wigner function $W_{EPR}(\alpha_1, \alpha_2)$ [6] of the (generalized) EPR state

$$W_{EPR}(\alpha_1, \alpha_2) = \frac{4}{\pi^2} \exp[-2 \cosh(2r)(|\alpha_1|^2 + |\alpha_2|^2) + 2 \sinh(2r)(\alpha_1 \alpha_2 + \alpha_1^* \alpha_2^*)] \quad (1)$$

is everywhere non-negative, it has been argued [3,6,8] that a local realistic interpretation of phenomena associated with the EPR state should be possible (here r is the parametric gain). On the other hand, Grangier, Potasek, and Yurke [9] have shown that, when homodyned against two weak coherent local oscillators (see Fig. 1), the EPR state exhibits a sinusoidal modulation of the photon counting rate as a function of the relative phases of the EPR and coherent states. They constructed a Bell-type inequality and showed that the violation of this inequality for the EPR state is the same as in Bohm-type experiments with two entangled spins [4].

More recently in a series of papers Banaszek and Wodkiewicz [10] have argued that negativity of the Wigner function is not required for violation of the Bell-type inequalities. In fact, they showed that the photon detection probabilities can be directly related to certain phase space densities and

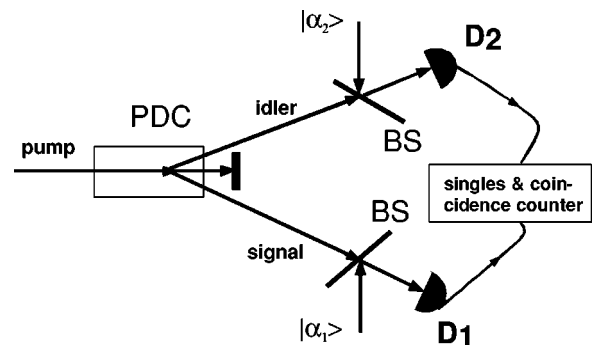


FIG. 1. Scheme to test Bell-type inequality for the EPR state. Photon pairs from a parametric down-converter (PDC) are mixed with weak coherent-state pulses at beam splitters (BS), and the numbers of photon counts are recorded at detectors D_1 and D_2 .

that for the EPR state in particular a Bell-type inequality can be violated. They made use of the observation that the Wigner function can be expressed as the expectation value of the displaced parity operator. Since the parity operator is a dichotomic observable, they were able to construct a Bell-type inequality similar to the one for two entangled spin-1/2 particles. The Banaszek and Wodkiewicz [10] inequalities are different from those of Grangier *et al.* [9], but they are also violated in a homodyne detection scheme of the type shown in Fig. 1. Moreover, although the degree of violation is different for the inequalities constructed by Grangier *et al.* [9] and by Banaszek and Wodkiewicz [10], the violation is most pronounced for small values of parametric gain r in both cases. The general principle behind these theoretical developments is that the Bell inequalities refer always to *the outcome of experimental measurements*. One cannot derive an inequality for a state alone, without considering the apparatus one is using to measure it. The experiments [6,7] were based on homodyning the EPR state against strong local oscillator fields and subsequent detection with linear photodiodes. By contrast, experiments proposed by Grangier *et al.* [9] and Banaszek and Wodkiewicz [10] require weak local oscillators and avalanche photodiodes in order to discriminate between the quantum mechanical and local realistic descriptions of nature. Both in our experiments and in those of Ou *et al.* [6], the measurement of the pair of single-mode fields is made by introducing a second mode at a beam splitter followed by detecting the total numbers of photons in the transformed modes. The detection is effectively the measurement of a *positive operator valued measure* (POVM) [11,12] where the auxiliary systems that extend the Hilbert space are the local oscillators. In the case of a strong coherent state local oscillator, as in the case of Ou *et al.*, the POVMs reduce to Hermitian quadrature field operators in the Hilbert space of the EPR state. In contrast, in our experiment no such reduction is possible. Therefore, unlike the experiment of Ou *et al.* [6], in our experiment the Wigner function cannot be used to formulate a local hidden-variable theory for the measurement. Hence, there is no contradiction between the existence of a local realistic description of the results of Ref. [6] and the violation of local realism that we observe. Our experiment provides experimental evidence contradicting Bell's assertion [3] that violation of the Bell inequalities in the EPR state is not possible. Bell based his conclusion on a restricted class of measurements, i.e., he considered only measurements of positions and momenta (or, equivalently, as in the case of Ou *et al.* [6], quadrature field amplitudes).

In this paper we describe an experiment that demonstrates the violation of a Bell-type inequality by light in an EPR state, earlier results of which were presented in Ref. [13]. The state was prepared by parametric down-conversion of ultrashort pump pulses. There are certain advantages to using short pulses to pump the PDC and for the local oscillators. In order to obtain high visibility of the interference it is vital that the local oscillator modes and the modes of the down-converted photons overlap in both space and time; it is difficult to achieve this with a narrowband cw laser, because of the long dead time of single-photon detectors and the need to maintain an adequate counting rate in the experiment. The

original proposal [9] treated only cw fields and therefore the analysis is not directly applicable to our experiment with ultrashort (~ 100 fs) pulses. We deal with the total number of detected coincidences instead of the time-correlation function and we include the effects of dispersion, and this distinguishes the pulsed light treatment from the cw treatment [9].

In our experiment we measure the observable corresponding to a coincident count between the detectors D_1 and D_2 as a function of the phases of the local oscillators ϕ_1 and ϕ_2 . We start by giving an intuitive single-mode derivation for the rate of coincident counts as a sinusoidal function of the phase difference θ between the down-converted light and the coherent pulse. In the single-mode approximation the EPR state for a small value of parametric gain r can be written as

$$|\Psi\rangle \approx |\text{vac}\rangle_s |\text{vac}\rangle_i + r e^{i\theta} |1\rangle_s |1\rangle_i. \quad (2)$$

We stress that, in contrast to the previous work demonstrating violations of local realism using biphotons, the two-photon part of the state *is factorizable*. After both the idler and the signal beams are mixed with weak local oscillators, so that

$$|\alpha\rangle_{1,2} \approx |\text{vac}\rangle_{1,2} + \alpha |1\rangle_{1,2} + \frac{\alpha^2}{2} |2\rangle_{1,2}, \quad (3)$$

the two-photon part of the two-mode quantum state at the detectors D_1 and D_2 is given by

$$|\Xi\rangle \approx \frac{1}{2} \alpha_2 \alpha_1 |1\rangle_1 |1\rangle_2 + \frac{r}{2} e^{i\theta} |1\rangle_1 |1\rangle_2 \quad (4)$$

(we omit terms of higher order in α_i , \sqrt{r}). The rate of coincidence counting $P(\alpha_1, \alpha_2)$ is given by

$$P(\alpha_1, \alpha_2) \equiv \langle \Xi | |1\rangle_1 \langle 1|_1 \otimes |1\rangle_2 \langle 1|_2 | \Xi \rangle, \quad (5)$$

and from Eqs. (4) and (5) we obtain

$$P(\alpha_1, \alpha_2) = \frac{r^2 + |\alpha_1|^2 |\alpha_2|^2}{4} [1 + V \cos(\phi_1 + \phi_2 - \theta)], \quad (6)$$

where $\phi_j \equiv \arg(\alpha_j)$, $j=1, 2$, and θ is the phase of the EPR state. The visibility V is given by $(2r|\alpha_1||\alpha_2|)/(r^2 + |\alpha_1||\alpha_2|)$. It reaches its maximum value of 1 when the coincidence rate due to coherent light equals the coincidence rate due to the EPR state.

Banaszek and Wodkiewicz [10] have shown that measurement of the expectation value of the displaced parity operator $\hat{D}(\alpha_1)(-1)^{\hat{n}_1} \hat{D}^\dagger(\alpha_1) \hat{D}(\alpha_2)(-1)^{\hat{n}_2} \hat{D}^\dagger(\alpha_2)$ in the arrangement of Fig. 1 gives the scaled Wigner function $(\pi^2/4)W(\alpha_1, \alpha_2)$ of the EPR state, while measurement of the expectation value of the displaced zero-count operator $\hat{D}(\alpha_1)|0\rangle_1 \langle 0|_1 \hat{D}^\dagger(\alpha_1) \hat{D}(\alpha_2)|0\rangle_2 \langle 0|_2 \hat{D}^\dagger(\alpha_2)$ gives the Q function. It is easy to see that the measurement of the probability $P(\alpha_1, \alpha_2)$ of detecting a coincident event between the detectors D_1 and D_2 is equivalent to measuring the difference between the Q and the scaled Wigner functions:

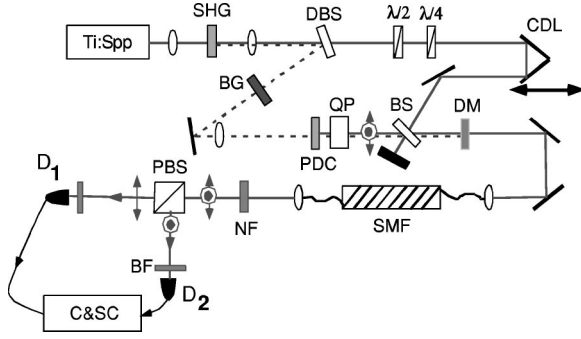


FIG. 2. Outline of the experimental setup. See text for details.

$P(\alpha_1, \alpha_2) = Q(\alpha_1, \alpha_2) - (\pi^2/4)W(\alpha_1, \alpha_2)$ if one keeps in mind that the $r^2, |\alpha_1|^2, |\alpha_2|^2$ are all much smaller than unity.

Grangier *et al.* [9] have constructed the following Bell-type inequality:

$$-2 \leq S \leq 2, \quad (7)$$

$$S \equiv E(\phi_1, \phi_2) - E(\phi_1, \phi'_2) + E(\phi'_1, \phi_2) + E(\phi'_1, \phi'_2),$$

where $E(\phi_1, \phi_2)$ depends on $P(\alpha_1, \alpha_2)$ for different sets of detectors and in our case is given by $E(\phi_1, \phi_2) = V \cos(\phi_1 + \phi_2 - \theta)$ (see [14] for a discussion). The violation of the inequality (7) is maximum for the following phase angles:

$$\begin{aligned} \phi_1 + \phi_2 - \theta &= \pi/4, \\ \phi'_1 &= \phi_1 - \pi/2, \\ \phi'_2 &= \phi_1 - \pi/2, \end{aligned} \quad (8)$$

with $S = V2\sqrt{2}$. Therefore, whenever $V > 1/\sqrt{2}$, $S > 2$ and the inequality (7) is violated. A discussion of supplementary assumptions relating to Bell-type inequalities in experiments based on homodyning techniques is given in Refs. [9,15,16].

II. EXPERIMENT

The experimental setup is shown in Fig. 2. The output power of the Ti:sapphire laser (Ti:Spp), consisting of a mode-locked train of pulses centered at 810 nm with bandwidth of 10 nm, is about 1.8 W on average. The light is used to pump a second harmonic generator (SHG) consisting of a 0.7 mm long β -barium borate (BBO) crystal, cut and aligned for collinear type-I phase matching. This creates a pulse of average wavelength 405 nm with about 15% energy conversion efficiency. The second-harmonic crystal is in effect the input beam splitter of our two-color interferometer. After the crystal both fundamental and second harmonic copropagate and are subsequently separated by a dichroic beam splitter (DBS). After passing through a red-light-blocking, blue glass filter (BG), the 405 nm light is focused onto the parametric down-converter (PDC), consisting of a 2.5 mm thick (BBO) crystal cut and aligned for type-II phase matching. It is found that, in order to effectively couple the down-conversion into a single-mode fiber (see below), we need to focus the 405 nm light into the PDC, so that the Gaussian waist of the light

in the crystal has a Rayleigh range approximately equal to the crystal length.

The orthogonally polarized, collinear down-converted photon pairs produced in the crystal are passed through an 8 mm thick quartz plate (QP), aligned so as to compensate for the relative delay suffered by the orthogonally polarized photons in the BBO crystal. They are then recombined on a 50:50 beam splitter (BS) with the red light which has been highly attenuated and passed through a controllable delay line (CDL) consisting of a translation stage and a piezoelectric-transducer (PZT)-mounted mirror. The CDL was calibrated by second-order interference with coherent light (a related experiment of homodyning *one* of the photons of the pair has been reported in [17]). The half-wave and quarter-wave plates allow us to change the relative phase $\phi_1 - \phi_2$ between the two coherent pulses. One of the two beam splitter outputs is passed through dichroic mirrors that reflect the 405 nm light and directed onto a polarizing Glan-Thompson beam splitter (PBS). The outputs of the PBS are focused onto the two avalanche photodiodes D_1 and D_2 acting as photon counting detectors. Broadband interference filters (BF) of 40 nm bandwidth are placed in front of the photodiodes, so as to reduce contributions from the scattered 405 nm radiation. Before the PBS, narrowband filters (NF) of different bandwidths are inserted in order to study the influence of dispersion (see below).

Photon detection events in the photon counting detectors produce output pulses that are passed through discriminators emitting 2 ns long voltage pulses. The outputs of the discriminators are fed into a coincidence AND gate, and coincident and single counts (C&SC) are registered. The 2 ns length of the pulses from the discriminators corresponds to a coincidence window of $\tau_c = 4$ ns. In the cw experiments the impetus for a short coincidence window comes from the fact that the rate of accidental coincident events that do not involve a down-conversion is given by the product of the single-photon counting rates and of the coincidence window or coincidence resolving time: $R_{acc} = R_1 R_2 \tau_c$. For the pulsed experiments, when the time separation between the pulses τ_r is larger than τ_c this expression becomes $R_{acc} = R_1 R_2 \tau_r$. For our 80 MHz repetition rate laser $\tau_r = 12$ ns, so that the number of accidental counts is not limited by the coincidence window.

Spatial wave front matching of the down-converted photons and of the coherent pulses is of critical importance for achieving good interference between them. The difficulty of spatial mode matching comes from the fact that, although the pump light going into the crystal has a Gaussian profile, the spatial profile of the down-converted “biphoton” output is given by the convolution of this Gaussian function with the phase-matching function of the crystal (see, e.g., [18]). This means that the photon pair is not well matched spatially to the Gaussian local oscillator. We therefore made use of a single-spatial-mode fiber to filter the interferometer output. This ensures very good spatial mode matching. With the help of the fiber, we were able to significantly increase the visibility of the interference pattern in our experiment. In addition to spatial mode matching, it is necessary to ensure correct temporal mode matching, in which the temporal profile of

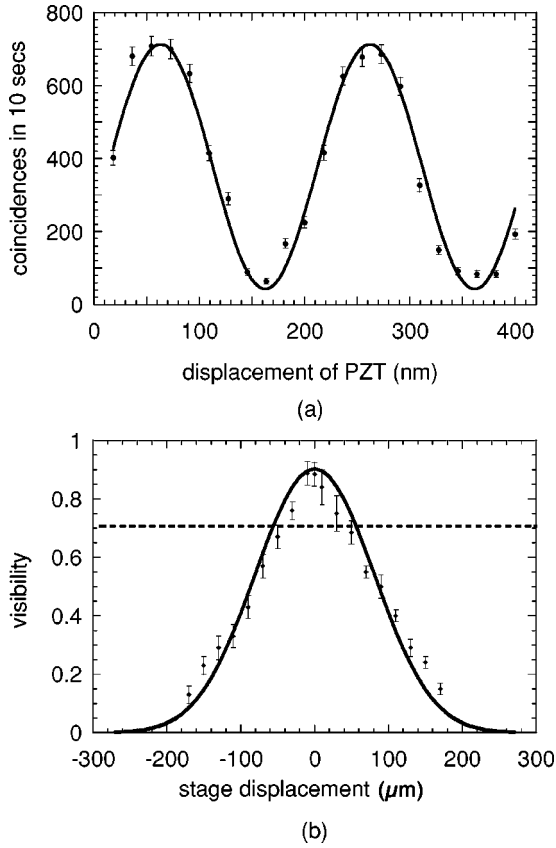


FIG. 3. (a) Interference pattern observed as a function of the displacement of PZT-mounted mirror with 3.5 nm filter in place (circles). The sinusoidal curve is the least squares fit based on the expression (6) when visibility, amplitude, and phase are treated as adjustable parameters, and is used to determine the observed visibility. (b) Diamonds correspond to the observed values of the visibility of the interference pattern as a function of the relative delay between the local oscillators and the down-converted photons. The solid curve is based on our theoretical calculation with no adjustable parameters. The dashed line corresponds to the highest visibility consistent with local realistic theories [9,14].

the field matches that of the single-photon wave packets. This is guaranteed to a large extent by using the same pulse to generate the down-converted pump radiation as for the local oscillators. The temporal modes of the down-converted radiation are also determined in part by the dispersive properties of the crystal, however, and we are not able to match the modes perfectly. However, with single-spatial-mode filtering, the measured visibilities were usually close to the calculated ones, even though they were a little less than unity due to dispersion in the down-converter.

We have recorded interference patterns as functions of $\phi_1 + \phi_2$ for four values of the narrow-band filter bandwidth: 3.5 nm, 6 nm, 16 nm, and 40 nm. Figure 3(a) shows the recorded interference pattern with the 3.5 nm filter at the maximum of the visibility curve according to Fig. 3(b). After flipping the half-wave plate and the quarter-wave plate we shifted $\phi_1 - \phi_2$ by π and $\pi/2$, respectively, and observed similar interference curves. In Fig. 3(a) the sinusoidal curve is the best (least squares) fit to the data when visibility, am-

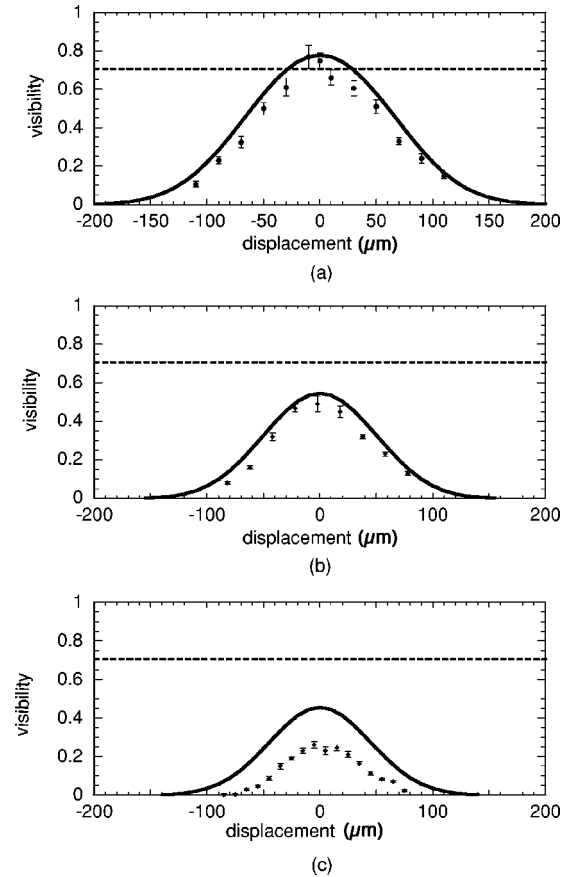


FIG. 4. Same as (b) in Fig. 3, but with 6 nm (a), 16 nm (b), and 40 nm (c) filter bandwidth.

plitude, and phase are treated as adjustable parameters. In Fig. 3(b) the visibility of the interference pattern as a function of displacement between the local oscillators and the EPR state is shown. The theoretical curve is based on the expression that we derived (see below) with no adjustable parameters. The visibility V reached $(89 \pm 4)\%$, so that $S_{exp} = 2.46 \pm 0.06$, in violation of the Bell inequality (7). Similar interference patterns, but with reduced visibilities, were observed with broader values of the filter bandwidth. The plots in Fig. 4 are similar to those in Fig. 3(b), but were taken with 6 nm (a), 16 nm (b), and 40 nm (c) filters. As the bandwidth of the filter increased, the visibility of the interference dropped markedly as shown by the experimental points in Fig. 5.

Our calculations (solid line in Fig. 5) show that both the walk-off between the pump pulse and the down-converted wave packet and the walk-off between the signal and idler photons resulting from the dispersion of the down-converter contribute to the reduction of visibility. We attribute the fact that the observed visibility of interference with the 40 nm filter is significantly lower than the theoretical value to the combination of possible birefringence in the optical elements and possible imperfections in the cancellation of signal-idler walk-off by the quartz plate, and to deviation of the filter transmission function from a true Gaussian. Nevertheless, the trend of increasing observed visibility with smaller filter bandwidth is apparent. This trend is due to the influence of

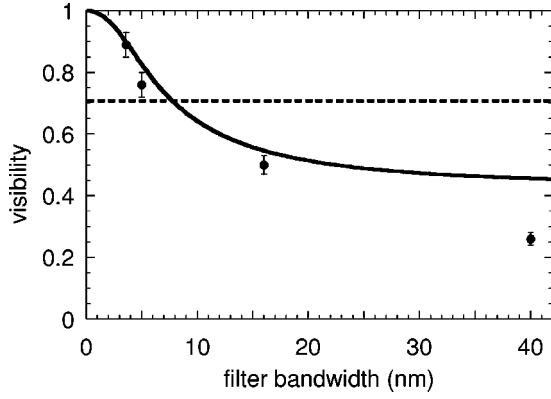


FIG. 5. Circles correspond to observed visibilities of the interference at their maxima for different values of the filter bandwidth. The solid line is obtained from our theoretical calculations and the dashed line has the same significance as in Fig. 3.

dispersion in the down-converter on the interference, as our calculations suggest.

The visibility of the interference pattern is significantly reduced if the signal-idler walk-off in the PDC crystal is not compensated because of improper temporal mode matching between the the single-photon wave packets and the local oscillator pulses. We have taken data with no quartz compen-

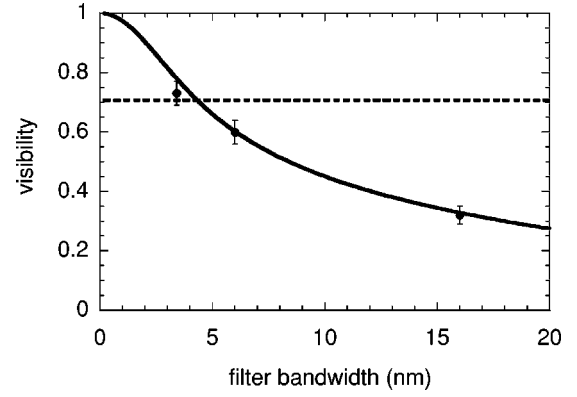


FIG. 7. Same as Fig. 5, but for situation with no signal-idler walk-off compensation.

sator for several filter bandwidths as shown in Fig. 6. The theory and the experimental points giving maximum visibility as a function of the filter bandwidth in the absence of signal-idler walk-off compensation are presented in Fig. 7.

Unlike the coincidence events, the single-photon counts in all four channels did not show any modulation at all as a function of ϕ_1, ϕ_2 , which lends support to the “no-enhancement” assumption of Grangier *et al.* [9].

III. THEORY OF HOMODYNE MEASUREMENTS WITH A PULSED EPR STATE

Consider the arrangement shown in Fig. 1: a nonlinear crystal functioning as a parametric down-converter is pumped by a short coherent pulse of light that generates two correlated light beams, a signal and an idler. Each of these beams is mixed with a weak coherent pulse on a 50:50 beam splitter and one of the outputs of each beam splitter is measured with a photon counting detector.

The theory of the down-conversion process has been treated many times. In our derivation we will use the approach of Refs. [14,19,20]. Let us consider a pump pulse centered at time $t=0$ with the corresponding electric field operator

$$\hat{E}_p(t) = \int_0^\infty d\omega \hat{a}_p(\omega_p) e^{-i\omega_p t}, \quad (9)$$

where $\hat{a}_p(\omega_p)$ represents the Fourier amplitude of the pumping pulse. Following Refs. [18,19,14], we can derive the following first approximation to the quantum state of the down-converted field:

$$\begin{aligned} |\Psi\rangle_{p,s,i} = & |\text{vac}\rangle_{p,s,i} + \eta \int_0^\infty \int_0^\infty d\omega_s d\omega_i \hat{a}_p(\omega_p) \\ & \times \exp(i\omega_p t) \Phi(\omega_p, \omega_s, \omega_i) \delta(\omega_p - \omega_s - \omega_i) \\ & \times \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |\text{vac}\rangle_{p,s,i}, \end{aligned} \quad (10)$$

where η is the parametric gain,

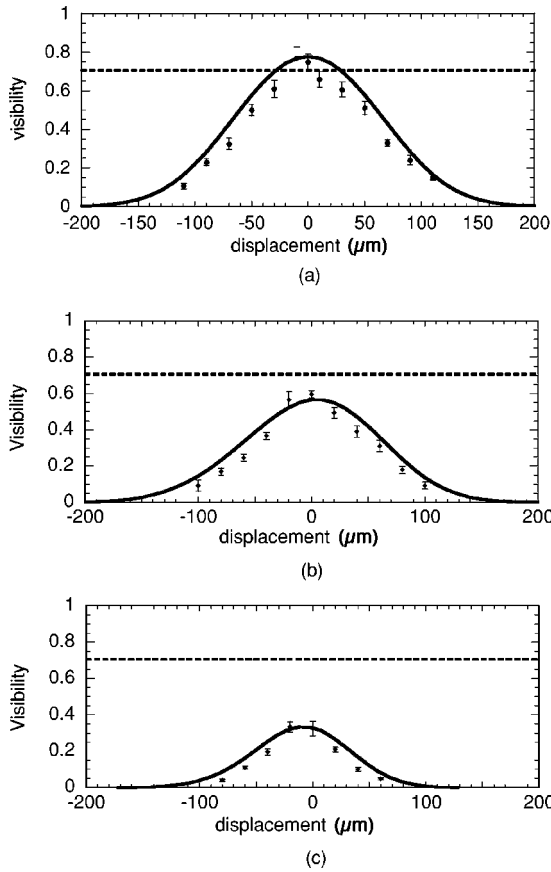


FIG. 6. Same as in Fig. 4 and Fig. 3(b) but without quartz compensator. (a) is taken with 3.5 nm filter, (b) with 6 nm filter, and (c) with 16 nm filter.

$$\Phi(\omega_p, \omega_s, \omega_i) \equiv C(\omega_s, \omega_i) \exp\left(\frac{i\Delta k L}{2}\right) \times \text{sinc}\left(\frac{i\Delta k L}{2}\right), \quad (11)$$

Φ is the phase matching function, L is the length of the crystal, and the normalization function $C(\omega_s, \omega_i)$ varies with frequencies ω_s, ω_i of signal and idler photons; $\Delta k \equiv k_p - k_s - k_i$ is the phase mismatch.

We have chosen the following normalization for $\Phi(\omega_p, \omega_s, \omega_i)$:

$$\int_0^\infty d\omega_s |\Phi(\omega_p, \omega_s, \omega_i)|^2 = 1. \quad (12)$$

In the Introduction some arguments based on single-mode calculations on the nature of the interference between the EPR state and LO pulses were given. Such arguments ignore the vast Hilbert space associated with the continuum of modes of different frequencies in the state (10). Some recent work of Law, Walmsley, and Eberly [21] provides a way to connect the single-mode treatment with the full QED calculation below. They showed with the help of the Schmidt decomposition that the two-photon part of the state (10) can be written as

$$|\Psi\rangle_{p,s,i}^{two\ photon} \approx \sum_{n=1}^{\infty} \sqrt{\lambda_n} \int \int d\omega_s d\omega_i \psi_n(\omega_s) \phi_n(\omega_i) \times \hat{a}_{ns}^\dagger(\omega_s) \hat{a}_{ni}^\dagger(\omega_i), \quad (13)$$

where λ_n , ψ_n , and ϕ_n are solutions of certain integral eigenvalue equations [21]. It is a remarkable property of the decomposition (13) that in situations of interest, e.g., in the experiment described above, only the lowest few values of n give noticeable contributions. In many cases the first term in the sum provides the dominant contribution. They also found that ψ_1 and ϕ_1 are approximately Gaussian functions of frequency, when a Gaussian pump pulse is used and when the crystal phase-matching function is spectrally broad.

We now have a direct connection between the single-mode arguments in the Introduction and the quantum state in the experiment. The visibility of the interference pattern provides an approximation to the inverse number of effective modes in the EPR state. Thus, when the observed visibility is 89% (see Fig. 3), about 90% of the EPR state is the same two-mode state.

We now derive some theoretical expressions to compare with the experimental data. For a crystal cut and aligned for degenerate type-II phase matching we have

$$k_p(\omega_{p0}) - k_s(\omega_{s0}) - k_i(\omega_{i0}) = 0, \quad (14)$$

where $\omega_{s0} = \omega_{i0} = \omega_{p0}/2$ are mid frequencies. On expanding Δk around the phase-matching condition (14), we obtain, to the first order in $\omega_s - \omega_{s0}$, $\omega_i - \omega_{i0}$,

$$\Delta k(\omega_s, \omega_i) = [k'_p - \frac{1}{2}(k'_s + k'_i)]\Omega_p + (k'_i - k'_s)\Omega, \quad (15)$$

where we have defined $\Omega_p \equiv \omega_p - \omega_{p0}$, $\Omega \equiv (\omega_s - \omega_i)/2$, and $k'_l \equiv (dk_l/d\omega_l)$ ($l=p, i, s$). This allows us to express the phase-matching function in Eq. (11) in the form

$$\Phi(\omega_p, \omega_s, \omega_i) = \frac{\sqrt{t_-}}{\pi} e^{i(t_- \Omega_p + t_+ \Omega)} \frac{\sin(t_- \Omega_p + t_+ \Omega)}{t_- \Omega_p + t_+ \Omega}, \quad (16)$$

where

$$t_- \equiv \left(k'_p - \frac{1}{2}(k'_s + k'_i)\right) \frac{L}{2} \quad (17)$$

is the difference between the group delay experienced by the pump pulse and the average of the group delays of the idler and signal photons, and

$$t_+ \equiv (k'_i - k'_s) \frac{L}{2} \quad (18)$$

is the temporal walk-off between the signal and idler photons, after traveling through half the length of the PDC.

Let us suppose that idler and signal fields are mixed with two coherent pulses on 50:50 beam splitters and a photon counting detector is located at the output of each beam splitter (see Fig. 1). The electric field operators after the beam splitters are then

$$\hat{E}_1(t) = \frac{1}{\sqrt{2}} [\hat{E}_{b_1}(t) + \hat{E}_i(t)], \quad (19)$$

$$\hat{E}_2(t) = \frac{1}{\sqrt{2}} [\hat{E}_{b_2}(t) + \hat{E}_s(t)], \quad (20)$$

and the input electric field operators are given by

$$\hat{E}_{s,i}(t) = \int_0^\infty d\omega \hat{a}_{s,i}(\omega) e^{-i\omega t},$$

$$\hat{E}_{b_1,b_2}(t) = \int_0^\infty d\omega \hat{b}_{1,2}(\omega) e^{-i\omega t}. \quad (21)$$

The state of the two-mode coherent field is $|B_1\rangle|B_2\rangle$, with

$$\hat{b}_{1,2}(\omega)|B_{1,2}\rangle = \beta_{1,2}(\omega)|B_{1,2}\rangle. \quad (22)$$

Before the photons impinge on the detectors, they pass through identical narrowband interference filters of bandwidth σ_f centered at ω_{s0}, ω_{i0} . Then the electric field operators at the detectors are given by [14,20]

$$\hat{E}_{D_1}(t) = \frac{1}{\sqrt{2}} \int_0^\infty d\omega [\hat{a}_i(\omega) + \hat{b}_1(\omega)] f_1(\omega) e^{-i\omega t},$$

$$\hat{E}_{D_2}(t) = \frac{1}{\sqrt{2}} \int_0^\infty d\omega [\hat{a}_s(\omega) + \hat{b}_2(\omega)] f_2(\omega) e^{-i\omega t}, \quad (23)$$

where $f_1(\omega), f_2(\omega)$ are the complex filter transmission functions. We will choose them to be identical and denoted $f(\omega)$ hereon. The probability of detecting a coincident event between the detectors D_1 and D_2 , $P(\beta_1, \beta_2)$, is given by

$$P(\beta_1, \beta_2) = \int_0^\infty \int_0^\infty dt_1 dt_2 \langle \Psi, B_1, B_2 | \hat{E}_1^-(t_1) \hat{E}_2^-(t_2) \times \hat{E}_2^+(t_2) \hat{E}_1^+(t_1) | \Psi, B_1, B_2 \rangle. \quad (24)$$

With help of Eq. (10) for the state $|\Psi\rangle$ we find that

$$P(\beta_1, \beta_2) = \frac{1}{4} \int_0^\infty \int_0^\infty dt_1 dt_2 | \eta g(t_1, t_2) + B_1(t_1) B_2(t_2) |^2, \quad (25)$$

where

$$g(t_1, t_2) = \int_0^\infty \int_0^\infty d\Omega_p d\Omega \alpha(\Omega_p) \Phi(\Omega_p, \Omega) \times f(\Omega_p/2 + \Omega) f(\Omega_p/2 - \Omega) \times \exp[-i(\Omega_p/2 - \Omega)t_1 - i(\Omega_p/2 + \Omega)t_2] \quad (26)$$

and

$$B_{1,2}(t) = \int_0^\infty d\omega f(\omega) \beta_{1,2}(\omega) e^{-i\omega t}. \quad (27)$$

After performing integrations over t_1, t_2 we obtain δ functions that reduce Eq. (25) to the form

$$P(\beta_1, \beta_2) = A + B + C. \quad (28)$$

The first term on the right is given by

$$A \equiv \frac{t_+ \eta^2}{4\pi} \int_0^\infty \int_0^\infty d\Omega_p d\Omega \alpha_p^2(\Omega_p) \Phi^2(\Omega_p, \Omega) \times f^2(\Omega_p/2 + \Omega) f^2(\Omega_p/2 - \Omega), \quad (29)$$

where the pump envelope is

$$\alpha_p^2(\Omega_p) = \frac{\langle n_p \rangle}{\sqrt{\pi} \sigma_p} e^{-(\Omega_p/\sigma_p)^2}, \quad (30)$$

and $\langle n_p \rangle$ is the average number of photons per pump pulse. We have chosen Gaussian transmission functions for the filters, with

$$f^2(\Omega) = e^{-(\Omega/\sigma_p)^2}. \quad (31)$$

After a somewhat lengthy calculation we arrive at the following result:

$$A = \frac{t_+ \eta^2 \langle n_p \rangle \sigma_f \sigma_k}{4\sqrt{2}\pi\sigma_p M} \left(\pi \operatorname{erf}(M) + \sqrt{\frac{\pi}{2M}} (e^{-2M^2} - 1) \right). \quad (32)$$

Here $\operatorname{erf}(x)$ is the error function, and σ_k and M are defined by the following relations:

$$\frac{1}{\sigma_k^2} = \frac{1}{\sigma_p^2} + \frac{1}{2\sigma_f^2} \quad (33)$$

and

$$M \equiv \sqrt{t_-^2 \sigma_k^2 + t_+^2 \sigma_f^2 / 2}. \quad (34)$$

Similarly, we find

$$C \equiv \int_0^\infty \int_0^\infty d\Omega_p d\Omega \alpha(\Omega_p) \Phi(\Omega_p, \Omega) \times \beta(\Omega_p/2 + \Omega) \beta(\Omega_p/2 - \Omega) \times f^2(\Omega_p/2 + \Omega) f^2(\Omega_p/2 - \Omega) \times \cos[\Omega_p(\tau_1 + t_-) - \Omega_p(\tau_2 + t_+) + \theta]. \quad (35)$$

We select Gaussian envelopes of the same width for both coherent pulses:

$$\beta_{1,2}(\Omega) = \frac{\beta_{1,2}}{\sqrt{\sigma_c} \sqrt{\pi}} \exp\left(-\frac{\Omega^2}{2\sigma_c^2}\right). \quad (36)$$

After evaluation of the integrals, we obtain

$$C = \frac{\sqrt{t_+ \langle n_p \rangle} \eta |\beta_1| |\beta_2|^2 \pi^{1/4}}{4\sqrt{\sigma_p}} \frac{\sigma_{eff} \sigma_{pe}}{\sqrt{t_-^2 \sigma_{pe}^2 + t_+^2 \sigma_{eff}^2}} \times \left[\operatorname{erf}\left(\frac{1}{2}[1 - (p_1 - p_2)I]M_1\right) + \operatorname{erf}\left(\frac{1}{2}[1 + (p_1 - p_2)I]M_1\right) \right] \times \exp\left(-\frac{p_2^2}{4G^2}\right) \cos(\phi_1 + \phi_2 - \theta), \quad (37)$$

where $\theta \equiv (\omega_{s0} + \omega_{i0})\tau_1$, $\phi_{1,2} \equiv \arg(\beta_{1,2})$,

$$J \equiv t_+^2 + t_-^2,$$

$$p_1 \equiv 1 + (t_- \tau_1 + t_+ \tau_2)/J,$$

$$p_2 \equiv (-t_+ \tau_1 + t_- \tau_2)/J,$$

$$M_1 \equiv \sqrt{t_-^2 \sigma_k^2 + t_+^2 \sigma_{eff}^2}. \quad (38)$$

$\tau_1 = (1/2)(t_i + t_s - t_{c1} - t_{c2})$ is the difference of propagation times between the signal and idler photons and the coherent pulses to their respective beam splitters, and $\tau_2 = t_s - t_i$ is the difference of propagation times between the signal and idler photons (it is assumed that the PDC is removed from the setup in these definitions of τ_1 and τ_2). We have introduced σ_{pe} , σ_{eff} , and I via the relations

$$\frac{1}{\sigma_{pe}^2} \equiv \frac{1}{2\sigma_p^2} + \frac{1}{4\sigma_f^2} + \frac{1}{4\sigma_c^2},$$

$$\frac{1}{\sigma_{eff}^2} \equiv \frac{1}{\sigma_c^2} + \frac{1}{\sigma_f^2}, \quad (39)$$

with

$$I = \frac{t_+ t_- (\sigma_{pe}^2 - \sigma_{eff}^2)}{t_-^2 \sigma_{pe}^2 + t_+^2 \sigma_{eff}^2}, \quad (40)$$

$$G = \frac{t_-^2 \sigma_{pe}^2 + t_+^2 \sigma_{eff}^2}{J^2 \sigma_{pe}^2 \sigma_{eff}^2}. \quad (41)$$

The second term in Eq. (28) is the easiest to evaluate, and we obtain

$$B = \frac{|\beta_1|^2 |\beta_2|^2}{4} \frac{\sigma_f^2}{\sigma_c^2 + \sigma_f^2}. \quad (42)$$

Now it is easy to see that the coincidence rate P is a sinusoidal function of the phase difference θ between the down-converted light and the coherent pulse:

$$P(\beta_1, \beta_2) = M[1 + V \cos(\phi_1 + \phi_2 - \theta)]. \quad (43)$$

The expression for the visibility follows from the derived expressions for A , B , and C . The theoretical curves in Figs. 2–7 are based on the expressions derived above.

IV. SUMMARY

In this paper we have studied interference between pairs of photons produced by spontaneous down-conversion and

weak coherent light by using photon counting techniques. The observed interference can be regarded as a manifestation of locality violations via the Bell inequalities, as was pointed out by Grangier *et al.* [9], and it is also a demonstration of nonlocality in phase space [10]. Our experiment provides evidence contradicting Bell's assertion [3] that violation of the Bell inequalities in the EPR state is not possible. However, Bell's assertion was based on a restricted class of measurements, i.e., he considered only measurements of positions and momenta (or, equivalently, as in the case of Ou *et al.* [6], quadrature field amplitudes).

In conclusion, we have observed interference between pairs of photons produced by spontaneous down-conversion and weak coherent light with the help of photon counting techniques. The interference can be regarded as a manifestation of locality violations via the Bell inequalities, as was pointed out by Grangier *et al.* [9], and it is also a demonstration of nonlocality in phase space [10]. The conclusion that local realism is violated requires additional supplementary assumptions [22], including the fair sampling assumption, as discussed by Granier *et al.* [9]. Moreover, for an unambiguous test of local realism, one would need to switch the phases of the local oscillators independently and at random times.

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