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Generation of arbitrary Dicke states in spinor Bose–Einstein condensates

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Abstract

We demonstrate that the combination of two-body collisions and applied Rabi pulses makes it possible to prepare arbitrary Dicke (spin) states as well as maximally entangled states by appropriate sequencing of external fields. © 2001 Published by Elsevier Science B.V.

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It is now widely recognized that the use of squeezed atomic states has the potential for substantial improvement in the sensitivity of atom interferometers. This realization has led to much theoretical and experimental work on schemes to realize atomic spin squeezing. Building upon the seminal work on photon number squeezing by Kitagawa and Yamamoto [1], Kitagawa and Ueda proposed and clarified the basic issues concerning the definition and preparation of spin squeezed states (SSSs) [2,3]. This was followed by studies of the production of squeezed atomic states [4–7], [1] the transfer of squeezing from incident squeezed light to an assembly of cold atoms [8,9], and the measurement of a collection of squeezed atoms [10–15]. So far, though, this work seems limited to the quantum state control of nondegenerate systems.

At the same time, Bose–Einstein condensates (BECs) have become almost routinely available in the laboratory. They provide us with large coherent ensembles of ultracold atoms which seem ideal to perform quantum state manipulation experiments from an exquisitely well-controlled initial state. In particular, recent experiments on two-component BECs in ^{87}Rb [16] have led to considerable work on the dynamics of the relative phase and number fluctuations of Bose condensates. There have been vigorous efforts on the issue of temporal phase coherence between two coupled

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[1] We note that recently, entangled state generation of spinor BECs has been considered in Ref. [6] and squeezing and entanglement of atomic beams has been considered in Ref. [7].

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Bose condensates. A number of workers have thought of this system as a prototypical Josephson junction between two weakly coupled superfluid systems.

While much of the theoretical work so far amounts to what are essentially semiclassical (mean-field) analysis [17–20] and studies involving small quantum corrections going beyond the mean-field Gross–Pitaevskii equation [17,21–25], some publications have gone past this framework. For instance, Villain et al. [21], Steel and Collett [25], and Javanainen and Ivanov [23] have studied dynamical aspects of the quantum state of a coupled BEC. There has also been intensive work on spin manipulation in the context of spinor BECs, motivated by the experimental work on ^{23}Na by the group at MIT [26–28]. Theoretical analyses, notably that of Law et al. have addressed the issue of collective spin properties in spinor BECs [29]. Recently, Sørensen et al. studied spin squeezing in a spinor condensate due to the internal nonlinear atom–atom interaction [30]. In this note, we extend these results by describing a quantum control technique which allows to prepare *arbitrary* Dicke spin states, as well as maximally entangled states by means of properly sequencing of external coupling fields.

To set the stage for the discussion and relate it to the physics of BECs, we first recall some basic properties of coherent spin states (CSSs) [4–36]. Consider a two-component condensate with N_i atoms in component i, and $N_1 + N_2 = N$. A general state of the system can be written as a superposition of number difference states

$$|\Psi\rangle = \sum_{N_2=0}^{N} c_{N_2} |N_1, N_2\rangle. \tag{1}$$

Alternatively, we may introduce the effective angular momentum quantum number $j = (N_1 + N_2)/2 = N/2$ and the z-projection quantum number $m = (N_2 - N_1)/2$ and reexpress the state $|\Psi\rangle$ in terms of angular momentum states $|j, m\rangle$. In particular, we have the correspondence

$$|j, -j\rangle = |N_1 = N, N_2 = 0\rangle \tag{2}$$

a state which can be thought of as the "ground state" of coherent spin states. Our goal is to construct arbitrary states of the system from this ground state $|j, -j\rangle$.

In a landmark paper, Kitagawa and Yamamoto [1] showed that photons could be number squeezed by making use of a Kerr nonlinear medium. Their procedure consisted of splitting a coherent beam, passing one of its components through a Kerr nonlinear medium, and interfering the resulting light with the other coherent beam in a Mach–Zehnder interferometer. The enhanced squeezing achieved in that case results from the fact that it involves a quartic four-wave process, while ordinary squeezing is a quadratic process. Following this work, Kitagawa and Ueda [2,3] showed that it was possible to likewise produce SSSs by making use of spin Hamiltonians quadratic in the spin operators. Our proposed scheme relies similarly on the existence of such a term in the Hamiltonian describing the dynamics of a coupled two-component condensate.

We consider for simplicity the so-called two-mode model describing the coupled two-component condensate. This approximation consists in neglecting all modes except the condensate modes. Its second quantized Hamiltonian is

$$\begin{aligned} H = {} & E_1 b_1^\dagger b_1 + E_2 b_2^\dagger b_2 + \frac{u_{11}}{2}(b_1^\dagger b_1^\dagger b_1 b_1) \\ & + \frac{u_{22}}{2}(b_2^\dagger b_2^\dagger b_2 b_2) + u_{12}(b_1^\dagger b_1 b_2^\dagger b_2) \\ & - \frac{1}{2}(\Omega b_1 b_2^\dagger + \Omega^* b_2^\dagger b_1), \end{aligned} \tag{3}$$

where b_i is the annihilation operator for component i,

$$E_i = \int \mathrm{d}^3 r\, \phi_i^*(\mathbf{r}) \left[\hat{\mathbf{p}}^2/(2m) + V_i(\mathbf{r})\right] \phi_i(\mathbf{r}) \tag{4a}$$

is the single-particle energy of mode i, $V_i(\mathbf{r})$ being the trapping potential, and

$$u_{ij} = \frac{4\pi\hbar^2 a_{ij}}{m} \int \mathrm{d}^3 r\, |\phi_i(\mathbf{r})|^2 |\phi_j(\mathbf{r})|^2 \tag{4b}$$

describes the two-body collisions in the condensate in the s-wave scattering approximation. Here a_{ij} is the scattering length for a two-body collision between an atom of ith component and that of jth component and $\phi_i(\mathbf{r})$ represents the condensate wave function for mode i. Finally

$$\Omega = \Omega_0 \int d^3 r \, \phi_1(\mathbf{r}) \phi_2^*(\mathbf{r}) \tag{5}$$

describes the strength of the linear coupling between components. The procedure to obtain the Hamiltonian (3) and the discussion of the validity of the two-mode approximation can be found for instance in Refs. [17,21,25,31]. We note that this same Hamiltonian has been previously considered by Scott et al. [32,33] in quantum studies of self-localization.

The analysis of Eq. (3) is greatly simplified by the introduction of the angular momentum operators

$$\hat{J}_x = \tfrac{1}{2}(b_1^\dagger b_2 + b_2^\dagger b_1), \tag{6a}$$

$$\hat{J}_y = \frac{i}{2}(b_1^\dagger b_2 - b_2^\dagger b_1), \tag{6b}$$

$$\hat{J}_z = \tfrac{1}{2}(b_2^\dagger b_2 - b_1^\dagger b_1), \tag{6c}$$

in terms of which Hamiltonian (3) may be rewritten as

$$H = \kappa \hat{J}_z^2 - \Omega_x \hat{J}_x - \Omega_y \hat{J}_y, \tag{7}$$

where we have introduced the effective nonlinear coupling

$$\kappa = \tfrac{1}{2}(u_{11} + u_{22}) - u_{12}, \tag{8}$$

and the real Rabi couplings $\Omega_x = \mathrm{Re}(\Omega)$, $\Omega_y = \mathrm{Im}(\Omega)$. Note that some control of the nonlinear parameter κ can be achieved through the proper engineering of the trapping potential, and hence of the condensate wave functions $\phi_i(\mathbf{r})$. Another way to adjust κ is through tuning the scattering lengths via Feshbach resonances, although this is not always easy to do in practice. However, the purpose of the present work is to show that spin squeezing can be controlled and manipulated with the external coupling fields instead of the internal nonlinearity of the condensate. In deriving (7), we have assumed that

$$E_1 - E_2 + (N - 1/2)(u_{11} - u_{22}) = 0,$$

a condition that can always be achieved by shifting the energy levels of the condensate components. If this condition is not fulfilled the Hamiltonian (7) contains an additional term proportional to $\hat{J}_z$.

Importantly for the following discussion, we observe that the independent temporal control of the Rabi pulses characterized by Ω_x and Ω_y is a well-established experimental tool. Depending on the particular system of interest, these pulses can be in the form of laser light, radio-frequency microwaves or magnetic fields.

With the Hamiltonian (7) at hand, we now demonstrate how an appropriate choice of the external coupling fields allows one to generate arbitrary Dicke states from the ground state $|j, -j\rangle$. We first observe that a general CSS $|\theta, \varphi\rangle$ can be created by rotating $|j, -j\rangle$ by the angle θ about the axis $\vec{n} = (\sin\varphi, -\cos\varphi, 0)$,

$$|\theta, \varphi\rangle = \exp[-i\theta(\hat{J}_x \sin\varphi - \hat{J}_y \cos\varphi)]\,|j, -j\rangle. \tag{9}$$

Starting from state $|j, -j\rangle$, switching on the coupling pulses and properly adjusting the strength of Ω_x and Ω_y, we can prepare any CSS. For the CSS $|\theta, \varphi\rangle$, the expectation value and the variance of operator $\hat{J}_z$ are

$$\langle \hat{J}_z \rangle = -j \cos\theta,$$

$$\langle \Delta \hat{J}_z^2 \rangle = \frac{j}{2} \sin^2\theta.$$

(Note that neither $\langle \hat{J}_z \rangle$ nor $\langle \Delta \hat{J}_z^2 \rangle$ will change after the coupling fields are turned off.) Following the usual convention we say that a j-spin state is *squeezed along* $\hat{J}_z$ if the state has the same $\langle \hat{J}_z \rangle$ but reduced $\langle \Delta \hat{J}_z^2 \rangle$ compared to the CSS $|\theta, \varphi\rangle$. Or we can define a parameter [2]

$$\xi_z = \frac{2j\langle \Delta \hat{J}_z^2 \rangle}{j^2 - \langle \hat{J}_z \rangle^2}. \tag{10}$$

A state is squeezed along $\hat{J}_z$ if $\xi_z < 1$.

Procedures to achieve spin squeezing in the equatorial plane $\langle \hat{J}_z \rangle = 0$ was previously proposed by Kitagawa and Ueda [2,3], and more recently by

[2] We note that this definition of ξ_z differs from those used previously [2–5]. We prefer it because of its utility in evaluating not only the spin squeezed states discussed in Refs. [2–5] but for evaluating the Dicke states discussed here. According to our definition, the Dicke-like states which exhibit strong squeezing exhibit reduced fluctuations in J_z as desired for precision measurement.

Law et al. [37]. We briefly recall them as a preparation for our general scheme. In the first scheme, a pulse is applied on the ground state $|j,-j\rangle$ along the y-direction at $t=0$. The duration of the pulse is assumed to be short enough that the effects of the nonlinear interaction are negligible during this time. This pulse aligns the spin vector along the negative x-axis, with the resultant state, $|J,-J\rangle_x$. After the pulse, the system evolves under the Hamiltonian $H_{\rm spin}=\kappa\hat{J}_z^2$ until a second short pulse is applied along x at $t=t_1$. At the end of this pulse, we have

$$\langle \hat{J}_z\rangle = 0, \tag{11a}$$

$$\langle \Delta\hat{J}_z^2\rangle = \frac{j}{2}\left\{1+\left(\frac{j}{2}-\frac{1}{4}\right)\times\left[\mathscr{A}-\sqrt{\mathscr{A}^2+\mathscr{B}^2}\cos 2(\alpha+\delta)\right]\right\}, \tag{11b}$$

where $\alpha=-\int\Omega_x\,\mathrm{d}t/\hbar$ is the strength of the second pulse and $\delta=\frac{1}{2}\tan^{-1}(\mathscr{B}/\mathscr{A})$ with

$$\mathscr{A}=1-\cos^{2(j-1)}(2\kappa t_1),$$

$$\mathscr{B}=4\sin(2\alpha)\cos^{2(j-1)}(\kappa t_1)\sin(\kappa t_1).$$

Optimal squeezing in $\hat{J}_z$ is achieved for $\alpha=-\delta$ and $|\kappa|t_1=(3/8)^{1/6}j^{-2/3}$, which leads to $\langle\Delta\hat{J}_z^2\rangle_{\min}\approx(1/2)(j/3)^{1/3}$.

As pointed out by Law et al. Ref. [37], an alternative approach can be realized by switching on a coupling field along the x-direction immediately after the first $\pi/2$ pulse along the y-direction, so that the system evolves under the Hamiltonian $H=\kappa\hat{J}_z^2-\Omega_x\hat{J}_x$. During the evolution following the control pulses, $\langle\hat{J}_z\rangle$ remains equal to 0 while $\langle\Delta\hat{J}_z^2\rangle$ can be significantly reduced at appropriate time. Fig. 1 shows the time evolution of the parameter ξ_z for both pulse sequences.

The essential point of both previous schemes is, as already mentioned, that $\langle\hat{J}_z\rangle$ remains equal to zero after the first $\pi/2$ pulse: If the first pulse is used to creates a CSS with $\langle\hat{J}_z\rangle=m_0\neq 0$, then following the above procedures, one would obtain a state squeezed along $\hat{J}_z$ but with $\langle\hat{J}_z\rangle=m_1\neq m_0$ (i.e. these two schemes do not allow for the creation of a squeezed state with any prescribed

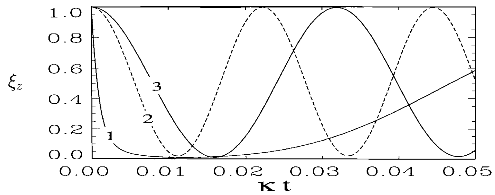

Fig. 1. Time evolution of the parameter ξ_z for $j=500$. Curve 1 corresponds to Eq. (11b) with $\alpha=-\delta$; the other two curves are obtained using the scheme of Ref. [30]. For curve 2, $\Omega_x/\kappa=20$; for curve 3, $\Omega_x/\kappa=10$.

$\langle\hat{J}_z\rangle=m_0\neq 0$). By using our procedure however, this need not be the case. In order to create squeezed states with arbitrary prescribed value of $\langle\hat{J}_z\rangle=m_0$ and arbitrary Dicke states, we propose to use instead the following method: first, a coupling pulse of appropriate strength is applied to the ground state $|j,-j\rangle$, creating an initial CSS with $\langle\hat{J}_z\rangle=m_0$. Assume without loss of generality that pulse is along the x-axis. Following this pulse, we then apply a coupling field along the negative x-axis. During the subsequent time evolution, both $\langle\hat{J}_z\rangle$ and $\langle\Delta\hat{J}_z^2\rangle$ start to oscillate. But at certain times when $\langle\hat{J}_z\rangle$ returns back to its initial value m_0, $\langle\Delta\hat{J}_z^2\rangle$ comes close to a local minimum which is less than the initial variance, i.e., $\xi_z<1$. Provided that the coupling field is turned off at these precise times, a state squeezed along $\hat{J}_z$ with $\langle\hat{J}_z\rangle=m_0$ will be created. For large enough squeezing, the state thus prepared can be regarded as an approximation of the Dicke state $|j,m_0\rangle$ which is the spin analog of the Fock state. A typical time evolution of $\langle\hat{J}_z\rangle$ and ξ_z is shown in Fig. 2(a) where the initial CSS has $j=500$ and $\langle\hat{J}_z\rangle=-250$. In this example, the optimal squeezing occurs at $\kappa t=0.0344$ where the variance $\langle\Delta\hat{J}_z^2\rangle$ is reduced by a factor of more than 20. Fig. 2(b) shows the probability distribution against the azimuthal number m at the optimal squeezing, while in Fig. 2(c) we plot the quasiprobability distribution (QPD) on a Bloch sphere.

Squeezing and entanglement are oftentimes closely related. In particular, it is well known that the quadratic Hamiltonian $H_{\rm spin}=\kappa\hat{J}_z^2$ also produces entanglement in a collective spin system. In particular, starting from the ground state $|j,-j\rangle_x$, $H_{\rm spin}$ generates at time $t^*=\hbar\pi/(2\kappa)$ the state

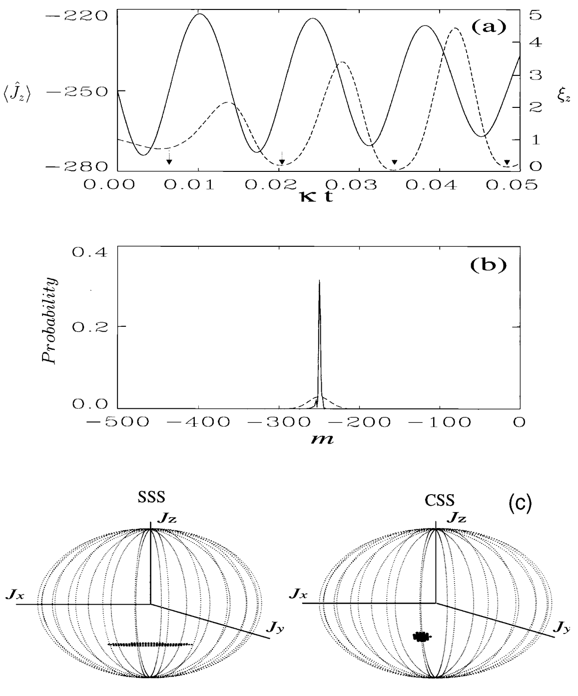

Fig. 2. (a) Time evolution of $\langle \hat{J}_z \rangle$ (left panel) and ξ_z (right panel). The initial CSS is $|\theta = \pi/3, \varphi = 0\rangle$ with $j = 500$. After the initial CSS is created, the system evolves under the Hamiltonian $H = \kappa \hat{J}_z^2 - \Omega_x \hat{J}_x$ with $\Omega_x/\kappa = -30$. At times indicated by the arrows, $\langle \hat{J}_z \rangle$ reaches the initial value (−250) while ξ_z is reduced. (b) Probability distribution of the azimuthal number m. (- - -) the initial CSS $|\theta = \pi/3, \varphi = 0\rangle$; (—) the spin squeezed state at $\kappa t = 0.0344$. (c) QPD on the Bloch sphere for the initial CSS and the squeezed state at $\kappa t = 0.0344$.

$$
\begin{aligned}
|\Psi(t^*)\rangle &= \exp\left(- \mathrm{i}\frac{\pi}{2}\hat{J}_z^2 \right) |j, -j\rangle_x \\
&= \frac{1}{\sqrt{2}} \left[\mathrm{e}^{-\mathrm{i}\pi/4} |j, -j\rangle_x + (-1)^j \mathrm{e}^{\mathrm{i}\pi/4} |j, +j\rangle_x \right].
\end{aligned}
\tag{12}
$$

The state is called *maximally entangled* because if one individual spin is found to be aligned along the negative x-axis or the positive x-axis, so are all other spins. However, the degree of entanglement degrades after t^*, a consequence of the fact that the state (12) is not an eigenstate of the Hamiltonian H_{spin}. To preserve the entanglement in our system, a $\pi/2$ pulse along the y-direction can be applied at $t = t^*$. This converts the state $|j, \pm j\rangle_x$ into $|j, \pm j\rangle$. The resultant state, still maximally entangled, it now an eigenstate of H_{spin}, hence it remains maximally entangled during the subsequent time evolution. This state is also a Schrödinger cat state since it is a coherent superposition of two macroscopically distinct states – one state has all the population in component 1 and the other in component 2.

Maximally entangled states, in particular those of massive particles instead of fast-escaping photons, are of great importance in fundamental physics as well as in applications in quantum information and quantum measurement. A great deal of effort has been directed toward the creation of such states. Two- [38,39], three- [40] and four-particle [41] entanglement have been successfully demonstrated experimentally in trapped ions, Rydberg atoms, and cavity QED. However, a further increase of the number of entangled particles in these systems are expected to be a severe experimental challenge. The two-component, atomic BEC with its built-in intrinsic nonlinearity appears to be a promising candidate to generate entanglement on a macroscopic level.

To conclude, we have shown that an arbitrary collective spin squeezing and entanglement of a two-component spinor condensate can be readily controlled by the coupling fields between the two components. Squeezed or entangled spin states will find applications in high-precision spectroscopy, atomic interferometry and quantum information, and spinor condensates are attractive candidates to create such states. In practice, the scheme presented in this work can be realized in the two-component ^{87}Rb condensate [16]. Another possibility is offered by the $F = 1$ spinor condensate of ^{23}Na [26–28] which can be reduced to an effective spin-$(1/2)$ system by shifting the energy level of the $m_F = 0$ state, e.g., with an ac Stark shift.

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