Editor's Note: Points similar to those made in the following two Comments were made in papers by S. Glasgow and J. Peatross, by M. Peshkin, by L. C. Cune and M. Apostol, by W. Luis Mochán and V. L. Brudny, and by C. Altucci, C. de Lisio, B. Preziosi, and S. Solimeno.

Comment on "Observation of Superluminal Behaviors in Wave Propagation"

In a recent work [1] it has been stated that experimental evidence has been found for superluminal propagation of a pulse of electromagnetic radiation. Since a number of earlier experiments claiming what might appear to be similar results are cited in [1], one might well be inclined to the view that this recent work is yet another confirming experiment. However, [1] is quite unique in its claims in that it does not require the presence of material media in order to demonstrate the effect. In other words, what is presented there is the very remarkable claim that superluminal wave propagation in vacuum is indeed possible. The object of this Comment is to question whether there is either any experimental or theoretical basis for such a conclusion.

Theoretical framework.—The experiment carried out in [1] is motivated by the existence of solutions of the Maxwell equations of the form [2]

$$A(\mathbf{x},t) = J_0(\rho k \sin\theta)\exp[i(zk\cos\theta - \omega t)], \quad (1)$$

where J_0 denotes the usual Bessel function, $k = \omega/c$, and ρ, z are cylindrical coordinates of $\mathbf{x}$. The angle θ can be chosen at will and has a possible interpretation as a cone angle. Reference [1] concludes from the form of the exponential in (1) that this implies propagation along the z axis at the superluminal velocity $c/\cos\theta$.

In order to display the flaw in this logic consider the plane wave $B_{\mathbf{k}}(\mathbf{x},t) = \exp[i\mathbf{k}\cdot\mathbf{x} - i\omega t]$, where $\mathbf{k} = (k\sin\theta\cos\phi, k\sin\theta\sin\phi, k\cos\theta)$. Although application of the methods of [1] would seem to imply superluminal velocity $c/\cos\theta$ along the z axis (and correspondingly along x and y), in this trivial example it is clear that $B_{\mathbf{k}}(\mathbf{x},t)$ is simply a wave traveling along the direction of $\mathbf{k}$ with velocity c. Thus the "true" (or group [3]) velocity component along z is $c\cos\theta$. However, once this is stated, it is straightforward to deal with the case (1) merely by recognizing that $A(\mathbf{x},t)$ can be obtained as a superposition of B waves having different wave vectors. Specifically, one performs a rotation of $\mathbf{k}$ about the z axis using the cone angle θ. From the identity

$$e^{i\mathbf{k}\cdot\mathbf{x}} = e^{izk\cos\theta}\sum_{-\infty}^{\infty} i^m e^{im(\phi_{\mathbf{x}}-\phi_{\mathbf{k}})}J_m(\rho k\sin\theta),$$

one readily finds

$$A(\mathbf{x},t) = \int_0^{2\pi}\frac{d\phi_{\mathbf{k}}}{2\pi}B_{\mathbf{k}}(\mathbf{x},t).$$

Thus the propagation velocity along the z axis of the wave $A(\mathbf{x},t)$ is clearly $c\cos\theta$ since it is composed entirely of waves whose velocity component along that axis is $c\cos\theta$.

Experimental situation.—A key aspect of the results in [1] is the ability of the apparatus to measure the speed of propagation of a pulse of microwave radiation with a carrier frequency of 8.5 GHz over a distance ranging from 0.30 to 1.40 m. Independent of the theoretical issues, the statistical significance of the data and the potential accuracy of the apparatus are subject to question. In particular, there is no proof that the apparatus is capable of measuring the propagation velocity with the needed accuracy. In absence of the elements used to create the Bessel beam, there are no data showing that the velocity measurement can be made at the accuracy required to make the 5% or 14% results significant. In a previous paper [4] two of the authors of [1] reported on microwave propagation measurements in a similar apparatus which did not use Bessel beams. In [4] they reported a time-of-flight accuracy of ± 0.1 ns for propagation over a distance of 0.90 m. From a simple analysis this confidence level appears reasonable inasmuch as in that experiment the microwave frequency corresponds to approximately 0.1 ns per cycle of the wave. Yet in [1] the accuracy required for the first measurement implies a confidence level which is comparable to or higher than that in [4], the justification for which is the improved performance of an oscilloscope used in the experiment. Systematic errors, either in time accuracy or in the conjugate variable of position accuracy, become less significant as the total time of flight (and hence the total path length) increases, exactly as in the data presented in [1].

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[1] D. Mugnai, A. Ranfagni, and R. Ruggeri, Phys. Rev. Lett. **84**, 4830 (2000).
[2] As in [1] the vector nature of the wave equation solutions will be ignored.
[3] A complete demonstration of this result would involve the construction of a wave packet using well known methods. It is absolutely crucial to note, however, that the "smearing" associated with this procedure must be done in both the radial and z coordinates. A superposition of waves of the form (1) with a weight factor which is a function of only a single variable (e.g., ω) will simply not give a localized beam.
[4] A. Ranfagni, P. Fabeni, G. P. Pazzi, and D. Mugnai, Phys. Rev. E **48**, 1453 (1993).

