Creation of topological states in spinor condensates

H. Pu,[1] S. Raghavan,[2,*] and N. P. Bigelow[3]
[1]Optical Sciences Center, The University of Arizona, Tucson, Arizona 85721
[2]Rochester Theory Center for Optical Science and Engineering and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
[3]Laboratory for Laser Energetics and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 13 June 2000; published 9 May 2001)

We present a simple scheme for generating topological phase states such as dark soliton and vortex states of a spinor Bose-Einstein condensate using applied magnetic fields. The realization of complex composite spinor vortices is described and we examine the subsequent dynamical evolution of these states.

DOI: 10.1103/PhysRevA.63.063603 PACS number(s): 03.75.Fi, 05.30.Jp

The soliton and vortex states are topological phase states that are characteristic solutions of nonlinear partial differential equations such as the nonlinear Schrödinger equation. These solutions have been well studied for the case of light propagation in a nonlinear medium and in the context of the Gross-Pitaevskii equations used to describe a Bose-Einstein condensate (BEC). Many aspects of these states have been studied experimentally in a variety of nonlinear optical systems and in the context of superfluid helium. After the successful generation of a trapped gaseous BEC, significant interest has turned to the generation of topological states in these macroscopic, matter-wave systems [1–5]. The recent observation of vortices in atomic BECs has also received significant attention because these vortices can be used to explore the relationship between the gaseous condensates and their superfluid counterparts, and because vortices serve as fundamental signatures of the superfluid character of Bose condensates. More generally, the creation and study of such topological phase states of a condensate are of fundamental interest because it represents a macroscopic system in which quantum state engineering can be developed and explored.

In this paper, we describe a simple method for creating topological states in a three-component spinor condensate using only static dc magnetic fields. The field couples the internal Zeeman sublevels of the condensate and ''imprints'' a topological phase onto the spinor condensate. The phase is determined completely by the geometric (spatial) structure of the magnetic field, is independent of the field strength, and takes full advantage of the complex internal dynamics—the spin-mixing effects—which characterize the spinor condensate. Because we consider a condensate with internal degrees of freedom, our method allows for the exploration of a broader class of topological phase states such as a complex vortex state in which vortex structures of the different spin components together form a single excitation.

One of the more straightforward means to create topological states in a BEC is the so-called *phase imprinting method*, in which, for example, the topological phase pattern of a coupling (laser) field is impressed onto the condensate. Using such a method, dark solitons have been created in single-component Rb [1] and Na [2] condensates and vortex states have been realized in a two-component Rb condensate [3,4]. There are also several proposals on the creation of solitons or vortices in spinor condensates [6–9] and on vortex generation in single-component condensates [10]. All these schemes rely on the presence of laser fields that have the proper wave configuration and hence provide the appropriate phase pattern and/or necessary coupling. In most of these schemes, the intensity and detuning of the laser fields, as well as the phase relationship among different fields, need to be carefully controlled in order to create the right phase. More recently, for the case of a spinor condensate, the use of combination of carefully chosen magnetic and laser fields has been proposed as a method for creating a toroidal-type trapping potential in which the magnetic-field geometry induces persistent currents, a situation that can lead to the creation of a vortex [11]. In our approach, only applied magnetic fields are considered. Moreover, our scheme provides a simple and straightforward experimental method for creating a rich class of states such as dark solitons and the complex spinor vortex state mentioned above.

Our model system is the well-studied $f=1$ spinor condensate [12] confined inside a harmonic trapping potential, $V_{\rm trap}=m[\omega_z^2z^2+\omega_\perp^2(x^2+y^2)]/2$, whose Hamiltonian, in the presence of external magnetic fields $\mathbf{B}$, may be written as $H=H_0+H_B+H_{\rm nl}$, where $H_0=\int d\mathbf{r}\,\hat\psi_\alpha^\dagger(\mathbf{r})(\hat T+V_{\rm trap})\hat\psi_\alpha(\mathbf{r})$ contains the kinetic and potential energy with $\hat\psi_\alpha$ being the boson field operator for spin state α ($\alpha=0,\pm1$; the identical subscripts are summed over), $H_B=-\int d\mathbf{r}\,\hat\psi_\alpha^\dagger(\gamma\mathbf{B}\cdot\mathbf{L}_{\alpha\beta})\hat\psi_\beta$ represents the interaction between the spinor condensate and the magnetic field with γ being the gyromagnetic ratio and $\mathbf{L}$ the angular momentum operator, and $H_{\rm nl}=\frac{1}{2}\int d\mathbf{r}\,\hat\psi_\alpha^\dagger\hat\psi_\beta^\dagger(c_0\delta_{\alpha\beta}\delta_{\mu\nu}+c_2\mathbf{L}_{\alpha\beta}\cdot\mathbf{L}_{\mu\nu})\hat\psi_\mu\hat\psi_\nu$ represents the internal nonlinear atom-atom interaction [13–15].

In this paper, we will show how to create one-dimensional dark solitons [16] and two-dimensional vortices by appropriately tailoring the spatial dependence of the time-independent magnetic field. We will also study the subsequent coherent time evolution of these states with and without the magnetic field. For convenience, we will assume throughout the paper that in the beginning ($t=0$), the spinor condensate is spin-polarized such that only the spin-0 component is populated, and that the spatial wave function is in its ground state as determined by the Gross-Pitaevskii equation [17,18].

*Present address: Corning, Inc., Corning, NY 14831.

1D solitons

For this case, we assume that the trapping potential is tightly confined in the radial (x-y plane) dimension such that the transverse motional degree of freedom is frozen. Hence a quasi-1D cigar-shaped condensate along the z axis will result. Further, we choose a magnetic field along the x axis whose strength linearly depends on z:

$$\mathbf{B}=B_0'(z-z_0)\hat{\mathbf{x}}. \tag{1}$$

Such a transverse field couples the spin-0 component to the spin-(± 1) components, and the spatial varying field strength twists the phase of the wave function such that phase jumps of π occur along the density profiles. These jumps are signatures of dark solitons.

To focus on the effect of the field, let us assume that the magnetic field is sufficiently strong such that the magnetic energy is much larger than all the other energy scales, and hence we can include only H_B in the Hamiltonian, neglecting all the other terms. This yields a set of linear differential equations:

$$i\dot{\psi}_0(z,t)=-b_x(z-z_0)[\psi_{-1}(z,t)+\psi_1(z,t)]/\sqrt{2},$$

$$i\dot{\psi}_{\pm 1}(z,t)=-b_x(z-z_0)\psi_0(z,t)/\sqrt{2},$$

where $b_x=\gamma B_0'/\hbar$. The above equations, under our initial conditions, can be easily solved to yield

$$\psi_0(z,t)=\psi_0(z,0)\cos[b_x(z-z_0)t],$$

$$\psi_{\pm 1}(z,t)=-(i/\sqrt{2})\psi_0(z,0)\sin[b_x(z-z_0)t].$$

Here $\psi_0(z,0)$ is the initial wave function for spin-0, which has a uniform phase along z. From this, one can see that whenever $\cos[b_x(z-z_0)t]$ (or $\sin[b_x(z-z_0)t]$) changes sign, there will be a precise π-phase jump in $\psi_0(z,t)$ (or $\psi_{\pm 1}$). Furthermore, the number of jumps (given roughly by $b_x t$ times the length of the condensate) increases with time, and, for spin-0, no phase jump ever occurs near the region $z=z_0$, while there is always a jump at $z=z_0$ for spin-(± 1). When we carry out numerical calculations using the full Hamiltonian, we find that most of these properties still hold regardless of the complexity of the problem. The only qualitative difference is that there is a maximum number of phase jumps (determined by the strength of the field) that can occur. This is due to the constraint of energy conservation, since increasing the number of jumps increases the kinetic energy. In fact, in the full calculation, the number of phase jumps is observed to oscillate in time.

Figure 1 shows the density profiles of the condensate at several different times when the B field is on. Here $z_0=0$, i.e., the zero field point coincides with the trap center. Also shown in the figure is the phase of the wave function, which has jumps of π radians at the nodes of the density profiles.

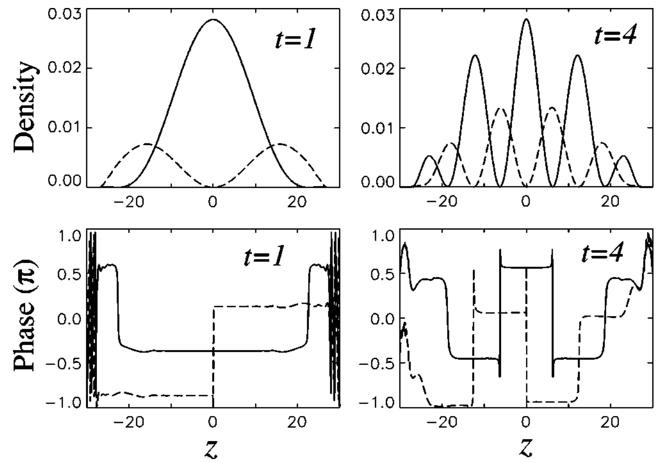

FIG. 1. Density and phase profile of the spinor condensate under transverse magnetic $\mathbf{B}=B_0'(z-z_0)\hat{\mathbf{x}}$. Solid lines: spin-0; dashed lines: spin-(± 1). In the phase plots, the fast oscillations at the ends are numerical artifacts. The parameters used correspond to a ^{23}Na spinor condensate with 2×10^5 atoms confined in a harmonic trap with $\omega_z=2\pi\times 40$ Hz, $\omega_\perp=4\pi\omega_z$. Here $B_0'=20$ mG/cm, $z_0=0$. The units for length and time in Figs. 1–4 are $\sqrt{\hbar/(2m\omega_z)}$ and $1/\omega_z$, respectively.

Figure 2 shows the population evolution under the effect of such a field. Collapse and revival structure can be observed due to the dephasing and rephasing of the high excitation modes induced by the field as well as the internal nonlinearity of the condensate.

Figure 3 shows the dynamics of the system at zero B field after one dark soliton has been created. For this case, the same field as in Fig. 1 is turned on from $t=0$ to 1 and then off afterwards. One can see from this example that at zero B field, the system experiences a periodic evolution as governed by its internal, nonlinear atom-atom interaction. At the end of each period, the system returns back to the state at the beginning of that period. Such a phenomenon is the matter wave analog of a high-order or periodic soliton state studied extensively in the context of nonlinear optics.

Figure 4 shows a similar evolution to that in Fig. 3 except that here the zero-magnetic-field point is away from the trap center, i.e., $z_0\neq 0$. Such a field creates a displaced dark soliton state in spin-(± 1), with the initial phase jump occurring at $z=z_0$. After the field is turned off at $t=1$, this phase singularity starts to move (in contrast to the case of a non-

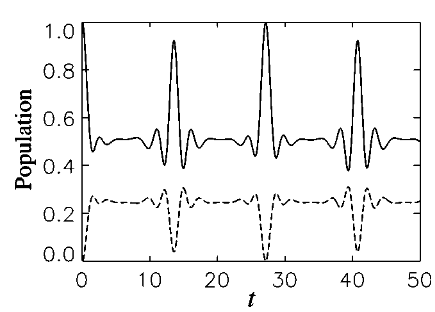

FIG. 2. Population evolution of the spinor condensate under $\mathbf{B}=B_0' z\,\hat{\mathbf{x}}$. Same parameters as in Fig. 1.

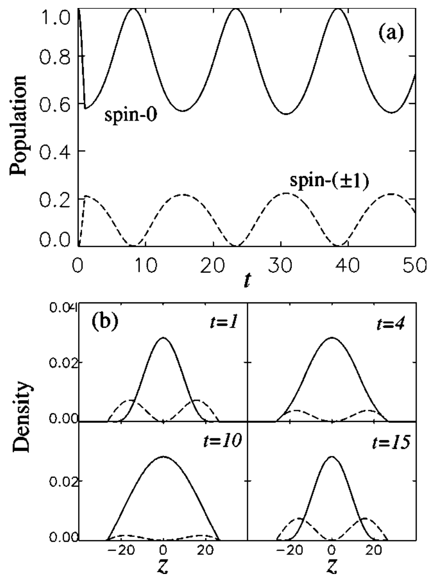

FIG. 3. (a) Population as functions of time. The field $\mathbf{B}=B_0' z\,\hat{\mathbf{x}}$ is on from $t=0$ to 1. (b) Density profiles for spin-(± 1) at several different times. Same parameters as in Fig. 1. Solid lines: spin-0; dashed lines: spin-(± 1).

displaced soliton, where the phase singularity at $z=0$ is stationary) to the edge of the condensate with increasing speed until it reaches the boundary of the condensate, when it reappears at the opposite end. After that, it moves towards the trap center with decreasing speed and stops at $z=-z_0$ before it reverses and moves back towards the edge again. This motion repeats itself and the phase singularity never enters in the region of $|z|<|z_0|$ [19]. Comparing the population evolution in zero field in Figs. 3 and 4, we see that in both cases the population oscillates with the same frequency [since the frequency is determined by the (same) internal nonlinearity]. However, the population oscillation amplitude decreases as $|z_0|$ increases. This can be intuitively understood: As $|z_0|$ increases, the constant part of the field $B_0' z_0$ becomes more dominant. Meanwhile, the effect of a constant transverse magnetic field on the initially spin-polarized condensate is to rotate the total spinor vector in spin space such that after the field is turned off, the spinor condensate is still in a stationary state without exhibiting population exchange among its components [18].

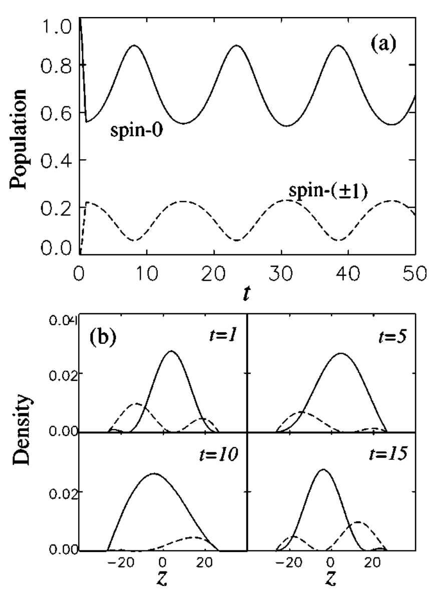

FIG. 4. (a) Population as functions of time. The field $\mathbf{B}=B_0'(z-z_0)\hat{\mathbf{x}}$ is on from $t=0$ to 1. (b) Density profiles for spin-(± 1) at several different times. Same parameters as in Fig. 1 except that $z_0=5$. Solid lines: spin-0; dashed lines: spin-(± 1).

2D vortices

If, contrary to the previous case, the longitudinal dimension of the condensate is tightly confined, then we will have a pancake-shaped quasi-2D condensate. If we then choose the magnetic-field configuration to be

$$\mathbf{B}=B_0'(x\hat{\mathbf{x}}-y\hat{\mathbf{y}}), \tag{2}$$

the magnetic coupling in matrix form may be written as

$$\mathbf{B}\cdot\mathbf{L}=\frac{B_0'\rho}{\sqrt{2}}\begin{pmatrix} 0 & e^{i\phi} & 0 \\ e^{-i\phi} & 0 & e^{i\phi} \\ 0 & e^{-i\phi} & 0 \end{pmatrix}, \tag{3}$$

where $\rho\equiv\sqrt{x^2+y^2}$ and ϕ is the polar angle in the x-y plane. One can see immediately from Eq. (3) that under such a

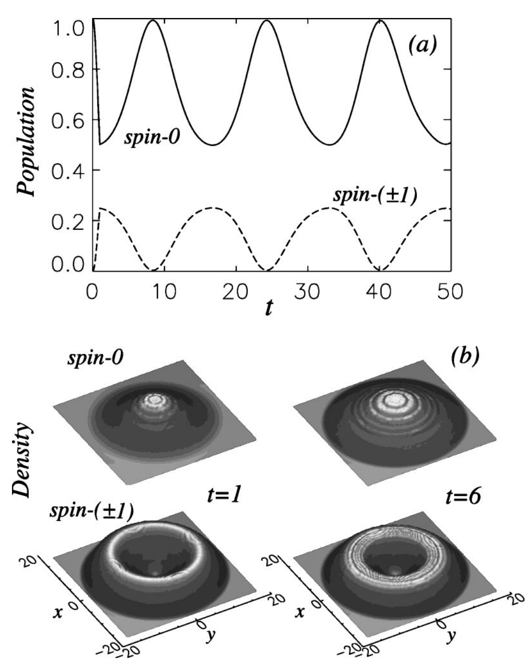

FIG. 5. (a) Population as functions of time. The field $\mathbf{B}=B_0'(x\,\hat{\mathbf{x}}-y\,\hat{\mathbf{y}})$ is on from $t=0$ to 1. (b) 2D density profiles at $z=0$ plane. The parameters used correspond to a ^{23}Na spinor condensate with 10^6 atoms confined in a harmonic trap with $\omega_\perp=2\pi\times 40$ Hz, $\omega_z=4\pi\omega_\perp$. Here $B_0'=25$ mG/cm. The units for length and time are $\sqrt{\hbar/(2m\omega_\perp)}$ and $1/\omega_\perp$, respectively.

field, the initial spin-0 condensate will be coupled to spin-(± 1) in the single vortex state. In practice, the field described in Eq. (2) can be obtained with four straight wires that carry currents $+I$ and $-I$ alternately and run parallel to the z axis and intersect the x-y plane at the corners of a square [20].

Figure 5 shows the generation and subsequent evolution of the vortex state. Again, the B field is turned off at $t=1$. The evolution is very similar to what is depicted in Fig. 3: From $t=0$ to 1, when the field is on, a vortex state is created in spin-(± 1); after the field is turned off, the system undergoes nonlinear oscillation under the internal interaction. We want to point out here that the spin states $+1$ and -1 have opposite angular momenta $\pm\hbar$, respectively, while the spin-0 state has zero angular momentum. Hence the spin and orbital angular momenta are entangled. Moreover, the vortex core size is widened by the mean-field repulsion from the spin-0 atoms, which occupies the interior region of the vortex. A similar effect is also observed in the two-component Rb system [4]. This feature may make it possible for the direct detection of the vortex state via density measurement.

With different field configurations, it is possible to create higher-order vortex states. To create vortices with winding number $\pm n$, one needs to have a magnetic field:

$$\mathbf{B}\propto \mathrm{Re}[(\tilde{x}-i\tilde{y})^n]\hat{\mathbf{x}}+\mathrm{Im}[(\tilde{x}-i\tilde{y})^n]\hat{\mathbf{y}},$$

where $\tilde{x}=x-x_0$ and $\tilde{y}=y-y_0$, with (x_0,y_0) being the position of the vortex core. For example, the Friedburg-Paul hexapole configuration with six wires can produce second-order vortex states [20]. It will be interesting to study the dynamical and stability properties of these systems.

In conclusion, we have proposed a scheme to create topological states in $f=1$ spinor condensates. Compared with other methods, ours possesses the following advantages: (i) It only involves dc magnetic fields; (ii) the topological phase imprinted on the condensate is precise (π for the dark soliton case and $2n\pi$ for the vortex case) as it arises solely from the geometric configuration of the field and does *not* depend on the strength of the field. Furthermore, the field configuration is simple enough to be easily realized in practice; (iii) the position of the phase singularity can be controlled in a straightforward manner.

In practice, one needs to account for the noise of the external field, i.e., the fluctuations in the strength and position of the field. The former does not pose a problem since, as we mentioned above, the topological phase depends only on the spatial geometry of the field. To estimate the effect of the pointing stability of the magnetic field, we ran our numerical code by adding a random noise to the position of the zero point of the field. From this study, we found that for high-frequency noise (frequencies on the order of or larger than $10\omega_z$ for the parameters used in this paper), the results above do not change significantly since in this case the condensate sees an averaged field with noises canceled out. On the other hand, for sufficiently strong low-frequency noise, the pointing stability of the field does play a role. Instead of creating a clean soliton or vortex state, it may create a state with many undesired topological defects. Hence, to implement our scheme in experiment, one must use a magnetic field with suppressed low-frequency pointing noise. This is, however, readily achievable with the current technology as low-frequency noise can be controlled via active stablization, and clever design can also force whatever noise there is to be more amplitude than pointing noise (i.e., by having the coils use common currents so any fluctuations are ''common mode'').

The authors would like to thank Dr. C. K. Law for seminal discussions which helped to inspire this work. This work is supported by the National Science Foundation, the Office of Naval Research, and by the David and Lucile Packard Foundation.

[1] S. Burger, K. Bongs, S. Dettmer, W. Ertmer, K. Sengstock, A. Sanpera, G.V. Shlyapnikov, and M. Lewenstein, Phys. Rev. Lett. **83**, 5198 (1999).
[2] J. Denschlag, J.E. Simsarian, D.L. Feder, C.W. Clark, L.A. Collins, J. Cubizolles, L. Deng, E.W. Hagley, K. Helmerson, W.P. Reinhardt, S.L. Rolston, B.I. Schneider, and W.D. Phillips, Science **287**, 97 (2000).
[3] J.E. Willians and M.J. Holland, Nature (London) **401**, 568 (1999).
[4] M.R. Matthews, B.P. Anderson, P.C. Haljan, D.S. Hall, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **83**, 2498 (1999); B.P. Anderson, P.C. Haljan, C.E. Wieman, and E.A. Cornell, *ibid.* **85**, 2857 (2000).
[5] K.W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000).
[6] K. Marzlin, W. Zhang, and E.M. Wright, Phys. Rev. Lett. **79**, 4728 (1997).
[7] R. Dum, J.I. Cirac, M. Lewenstein, and P. Zoller, Phys. Rev. Lett. **80**, 2972 (1998).
[8] J. Ruostekoski, Phys. Rev. A **61**, 041603(R) (2000).
[9] K. Marzlin, W. Zhang, and B.C. Sanders, e-print cond-mat/0003273.
[10] Ł. Dobrek, M. Gajda, M. Lewenstein, K. Sengstock, G. Birkl, and W. Ertmer, Phys. Rev. A **60**, R3381 (1999).
[11] T. Isoshima, M. Nakahara, T. Ohmi, and K. Machida, Phys. Rev. A **61**, 063610 (2000).
[12] D.M. Stamper-Kurn, M.R. Andrews, A.P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. **80**, 2027 (1998).
[13] T.L. Ho, Phys. Rev. Lett. **81**, 742 (1998).
[14] T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998).
[15] C.K. Law, H. Pu, and N.P. Bigelow, Phys. Rev. Lett. **81**, 5257 (1998).
[16] In this paper, we use the term soliton to refer to a solitary-wave excitation of the condensate. As noted in Ref. [2], collisions of the excitations are not considered. In this definition, for example, the trapped condensate ground state is not classified as a soliton.

[17] H. Pu, C.K. Law, S. Raghavan, J.H. Eberly, and N.P. Bigelow, Phys. Rev. A **60**, 1463 (1999).
[18] H. Pu, S. Raghavan, and N.P. Bigelow, Phys. Rev. A **61**, 023602 (2000).
[19] The speed of the phase singularity increases as it approaches the edge of the condensate where the local sound velocity is small due to the low atomic density. Hence it will be interesting to see whether this will induce dissipations in the system.
[20] E.A. Hinds, and C. Eberlein, Phys. Rev. A **61**, 033614 (2000).