

# Photon counting with a loop detector

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We propose a design for a photon-counting detector that is capable of resolving multiphoton events. The basic element of the setup is a fiber loop, which traps the input field with the help of a fast electro-optic switch. A single weakly coupled avalanche photodiode is used to detect small portions of the signal field extracted from the loop. We analyze the response of the loop detector to an arbitrary input field and discuss both the reconstruction of the photon-number distribution of an unknown field from the count statistics measured in the setup and the application of the detector in conditional-state preparation. © 2003 Optical Society of America

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Photon counting is an important method of detecting light and has applications in many fields including spectroscopy, atmospheric physics, and quantum information processing. Detection of single photons necessarily involves a gain mechanism to generate a macroscopic number of photoelectrons from the absorption of a single energy quantum from the incident electromagnetic radiation. Among the most popular detectors with this characteristic are avalanche photodiodes operated in the Geiger mode.<sup>1–3</sup> Compared with standard photomultipliers, they have higher quantum efficiency, are more stable, and are more robust with respect to external conditions.

Geiger-mode avalanche photodiodes have one drawback: The breakdown current is almost completely independent of the number of absorbed photons. Consequently it is not possible to determine the number of photons that are incident upon the detector on a time scale that is short compared with the detector's response time. This disadvantage has serious implications for several applications. For example, it critically affects the performance of practical quantum cryptography systems.<sup>4,5</sup> It also impairs the fidelity with which entangled states may be prepared<sup>6,7</sup> as well as the conditional detection required for linear optical quantum computing.<sup>8</sup>

In this Letter we propose a photon-counting setup that is capable of resolving multiphoton events. The basic idea is to split the input signal into separate small pieces that are expected to contain at most one photon and therefore can be detected with an avalanche photodiode without loss of information on the photon number. Our method is based on current fiber optic technology and has the important advantage that it requires only a single avalanche photodiode, in contrast with previous proposals that involve the splitting the signal input and therefore require a large array of photodiodes.<sup>9</sup> We note that multiphoton absorption events can in principle be resolved by more exotic photon-counting detectors such as solid-state photomultipliers.<sup>10</sup> These detectors, however, remain in the development stage and must be operated at liquid-helium temperatures.

The proposed loop detector is depicted schematically in Fig. 1. The basic element of the setup is a fiber

loop used as a storage cavity. The light pulse whose photon statistics are to be measured enters the loop through a fast electro-optic switch (S).<sup>11</sup> Immediately after injection of the pulse the state of the switch is flipped, which closes the loop and thus traps the input pulse. In each circulation of the loop, a small portion of the pulse is extracted by a weak static coupler (C) and is incident upon an avalanche photodiode (APD) operated in the Geiger mode. The duration of a single round trip in the loop is longer than the dead time of the APD, so after each round trip the APD is ready for a new detection event. The quantity recorded in the measurement is the total number of counts on the photodiode.

Our main purpose in this Letter is to quantify the relationship between the statistics of counts on the APD and the photon distribution of the input pulse. First we calculate the response of the detector to a coherent field. This result can be generalized to an arbitrary input field by averaging with an appropriate probability distribution for the field amplitude. In the most general case this probability distribution is given by Glauber's  $P$  representation for the input field. Next, we use this result to derive a method for reconstructing the photon-number distribution from the count statistics for a completely unknown input field.

We begin by calculating the probability distribution  $p_I(k)$  of obtaining  $k$  counts on the photodiode, assuming that the coherent pulse after injection into the loop

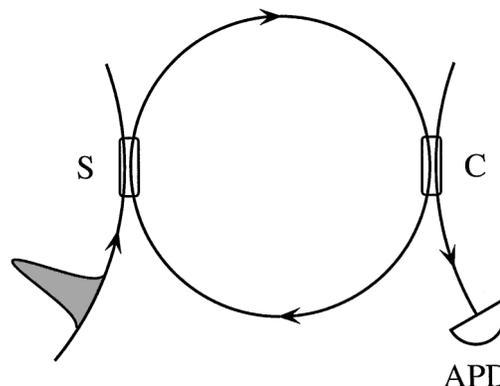


Fig. 1. Proposed configuration of the loop detector.

has intensity  $I$ . In the derivation it will be convenient to use the generating function

$$\tilde{p}_I(z) = \sum_{k=0}^{\infty} z^k p_I(k). \quad (1)$$

The assumption of coherent input substantially simplifies the calculations, because the portion of the pulse directed to the APD in each round trip is uncorrelated with the field that remains in the loop. Consequently, the counts registered by the APD in each round trip are statistically independent. Let  $t_r$  be the power transmission of the loop for a single pulse round trip, including losses at all the optical elements, and  $t_c$  be the fraction of the light intensity extracted by the coupler to the APD. In the  $i$ th circulation of the loop, the intensity of the pulse is  $t_r^{i-1}I$ . Of this, the light intensity extracted to the photodiode is  $t_c t_r^{i-1}I$ . If the efficiency of the APD is taken to be  $\eta$ , the probability of a count in the  $i$ th round trip is  $1 - (1 - p_d)\exp(-\eta t_c t_r^{i-1}I)$ , including the possibility of a dark count with probability  $p_d$ . For an input pulse in a coherent state the counts on the APD are statistically uncorrelated on each circulation, so the generating function  $\tilde{p}_I(z)$  is simply given by a product of generating functions that describe detection events for each round trip:

$$\tilde{p}_I(z) = \prod_{i=1}^L [z + (1 - z)(1 - p_d)\exp(-\eta t_c t_r^{i-1}I)], \quad (2)$$

where  $L$  is the total number of circulations of the loop.

The expression derived in Eq. (2) describes the most general situation of arbitrary input intensity. An important limiting case occurs when the probability of extracting two or more photons from the loop in a single round trip via the coupler is negligible. Then the logarithm of the generating function  $\ln \tilde{p}_I(z)$  can be expanded up to terms that are linear in  $t_c I$  or  $p_d$ , assuming that both of these parameters are much smaller than 1. Further, when the number of round trips is large enough to allow all the input signal photons to leak from the loop, we obtain from the logarithmic expansion the following approximate expression for the generating function:

$$\tilde{p}_I(z) \approx \exp\left[(z - 1)\left(\frac{\eta t_c I}{1 - t_r} + L p_d\right)\right]. \quad (3)$$

This formula describes simply the standard Poissonian statistics normally associated with coherent radiation, with the average number of counts equal to  $\eta t_c I / (1 - t_r) + L p_d$ . The first term here describes the average number of counts generated by the input light, and the second term corresponds to dark counts. In this regime the response of the loop detector is described by a single effective efficiency parameter:

$$\eta_{\text{eff}} = \eta \frac{t_c}{1 - t_r}. \quad (4)$$

In the ideal limit all the optical elements are lossless, and the only source of attenuation for the pulse trapped inside the loop is extraction of the photons

to the APD. Then  $t_c = 1 - t_r$ , and the effective efficiency of the loop detector approaches that of the APD but with the advantage of having photon-number resolution. In a realistic case there are always excess losses in the coupler or in the electro-optic switch, which decrease the effective efficiency of the loop detector below the level of the APD itself. This figure, multiplied by the coupling efficiency of the signal field to the input port of the loop detector, gives the overall detection efficiency.

We now turn to the problem of reconstructing the photon-number distribution of an unknown input field from the statistics of counts measured with the loop detector. Let  $\varrho(n)$  be the photon-number distribution of the pulse immediately after injection into the fiber loop. The probability  $p(k)$  of observing  $k$  photocounts is given by the linear combination

$$p(k) = \sum_n w(k|n)\varrho(n), \quad (5)$$

where  $w(k|n)$  are conditional probabilities of registering  $k$  counts on the photodiode, given exactly  $n$  photons injected into the loop. Their explicit form can be found from our previous calculation of the generating function for a coherent input [Eq. (2)]. For a coherent input pulse the photon-number distribution is Poissonian,  $\varrho(n) = e^{-I} I^n / n!$ . Inserting this formula into Eq. (5), multiplying both sides by  $z^k$ , and performing the summation over  $k$  yields

$$e^I \tilde{p}_I(z) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} w(k|n) \frac{z^k I^n}{n!}, \quad (6)$$

where  $\tilde{p}_I(z)$  is defined in Eq. (2). Thus we can identify the conditional probabilities  $w(k|n)$  from the generating function  $e^I \tilde{p}_I(z)$  for a coherent input by expanding the function into a double power series in the parameters  $I$  and  $z$ . This procedure can easily be performed by use of any standard computer program for symbolic algebraic calculations. Given the explicit form of the conditional probabilities  $w(k|n)$  and the measured count statistics  $p(k)$ , one can then use one of the standard methods for solving linear systems to reconstruct photon-number distribution  $\varrho(n)$ . In Fig. 2 we show the results of a Monte Carlo simulation of the loop detection operation in which the singular-value decomposition method<sup>12</sup> was used to reconstruct the photon-number distribution. The detector is able to provide good estimates of the photon-number distribution of both classical and nonclassical inputs. The accuracy of the reconstruction can be estimated with standard statistical tools.<sup>13</sup>

In a number of quantum information processing protocols,<sup>6-8</sup> determination of the photon number has to be made on a single-shot basis to permit nondestructive inference of the state of another quantum system that is entangled with the detected radiation. In such an application the performance of the detector can be characterized by the confidence<sup>9</sup>

$$C_k = \frac{w(k|k)\varrho(k)}{\sum_n w(k|n)\varrho(n)}, \quad (7)$$

which describes the *a posteriori* probability that a  $k$ -click event has been triggered by the  $k$ -photon

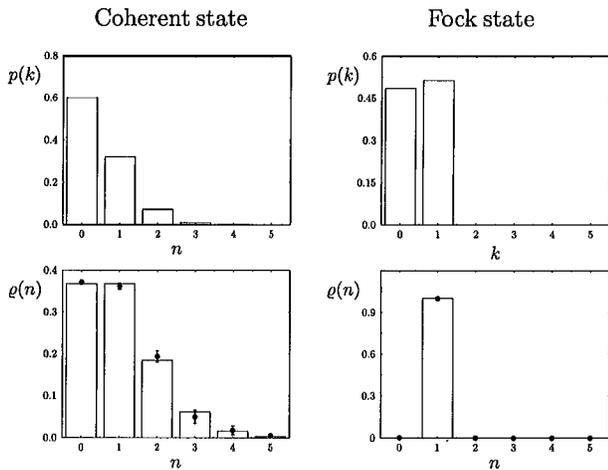


Fig. 2. Monte Carlo simulation of photon counting by use of a loop detector for a coherent state with the average photon number equal to 1 and for a single-photon Fock state. The top figures depict simulated count statistics based on  $N = 10^5$  repetitions of the measurement, assuming that  $t_r = 72\%$ ,  $t_c = 20\%$ ,  $\eta = 80\%$ , and  $p_d = 0$ . The filled circles in the bottom figures present the photon-number distributions reconstructed with the help of the singular-value decomposition method, compared with the exact distributions (shown by bars).

component of the input field state. Here  $q(n)$  is the *a priori* photon-number distribution of the input field and  $w(k|n)$  are the conditional probabilities calculated above. In practice, the confidence can be optimized by tuning of the splitting ratio of the coupler used in the construction of the loop detector.

An important question is whether realistic values for the parameters of the loop detector enable it to function in the manner shown in Fig. 2. In the visible region, silicon APDs can exhibit efficiencies greater than 75% for wavelengths near 700 nm.<sup>14</sup> The dark-count rates for these devices can be as low as 25 Hz, which is negligible. This rate may be further reduced by operation of the photodiode in the gated mode. The APD parameters become less favorable beyond the wavelength region about 1  $\mu\text{m}$ , where germanium or InGaAs materials are used.<sup>15,16</sup> As with single-photon detection, the quantum efficiency of the photodiode imposes the upper limit on the performance of the loop detector. A second important factor is the excess losses inside the fiber loop, which attenuate the pulse in each round trip in addition to the fraction extracted by the coupler. These excess losses lead to two opposing constraints on extraction of the light from the loop to the photodiode, parametrized by coupling losses  $t_c$ . On the one hand, to recover the complete photon-number distribution,  $t_c$  should be as small as possible. On the other hand, if too small a power is extracted from the loop, a large number of round trips will be required for measuring the count statistics. Consequently, substantial excess losses in the loop would mean that most of the photons would disappear inside the loop before ever reaching the photodiode and would give no signal at all. The optimal value of  $t_c$  is some intermediate value that extracts enough power of the trapped pulse to the APD and

at the same time provides a sufficiently large sample of multiple detection events, when multiphoton terms of the input photon distribution are chopped into a temporal sequence that contains at most one photon in each slot. A good test of whether both of these conditions are satisfied is to check the singularity of the matrix composed of the conditional probabilities  $p(k|n)$  for the range of photon numbers that are expected in the input signal. If the matrix is not singular, then the count statistics provide sufficient information with which to reconstruct the complete input photon-number distribution from the experimental data. Similar considerations can be applied to optimize the confidence, defined in Eq. (7), in the problem of conditional-state preparation.

In conclusion, we have proposed and analyzed a design for a photon-counting detector that is capable of resolving multiphoton detection events by use of standard laboratory technology. The setup uses commonplace fiber-optic components and does not require extreme operating conditions.

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