

carries no net spin, i.e., the up and down spin components of the pumped current, I_\uparrow and I_\downarrow , are identical. In this case, I_\uparrow and I_\downarrow fluctuate *together*, with zero average, as a function of external parameters, such as static dot shape and perpendicular magnetic field. Spin degeneracy can be lifted by applying a magnetic field in the plane of the 2DEG [14]. For moderate parallel fields, $E_Z = g^* \mu_B B_\parallel > \max\{\Delta, \hbar\gamma_{\text{esc}}, k_B T\}$ (typically a few Tesla for a micron scale GaAs quantum dot at temperatures below 0.5 K) the pumped currents associated with the two spin directions I_\uparrow and I_\downarrow will become uncorrelated, and will fluctuate independently as device parameters are changed.

Let us denote the charge and spin currents passing through the dot as I_c and I_s , respectively: $I_{c,s} = I_\uparrow \pm I_\downarrow$ (we define spin current to have the same units of charge current, understanding that $e \leftrightarrow \hbar/2$). Upon averaging over different realizations of the dot shape or chemical potential, $\overline{I_\uparrow} = \overline{I_\downarrow} = 0$. The strength of the pumping current is characterized by its variance,

$$\overline{I_{c,s}^2} = \overline{I_\uparrow^2} + \overline{I_\downarrow^2} \pm 2\overline{I_\uparrow I_\downarrow} = 2(\overline{I_\uparrow^2} \pm \overline{I_\uparrow I_\downarrow}), \quad (1)$$

where we assumed $\overline{I_\uparrow^2} = \overline{I_\downarrow^2}$. In the absence of an in-plane field, $\overline{I_\uparrow I_\downarrow} = \overline{I_\uparrow^2}$, whereas for a strong enough applied field, we expect that incoming spin up and spin down electrons will occupy uncorrelated sets of states in the dot, leading to $\overline{I_\uparrow I_\downarrow} = 0$. As a result, $\overline{I_c^2}$ decreases by a factor of 2 in the large field limit [15], while simultaneously $\overline{I_s^2}$ goes from zero to its maximum value. The most striking situation, however, occurs when parameters are set such that $I_\uparrow = -I_\downarrow$. Because I_c fluctuates randomly about zero as a function of external parameters, one can simply tune dot shape or perpendicular field until the condition $I_c = 0$ is found. This will be the state where $I_\uparrow = -I_\downarrow$. In this case, a finite spin current exists through the quantum dot without any net charge transport. Experimentally, gate voltage control at the level of tens of microvolts is sufficient to achieve this condition to within the noise of the pumped current. This is illustrated in Fig. 2.

It is important to realize that the effect of the Zeeman field is not to polarize the electrons in the dot, but rather to create two independent electron ‘‘speckle patterns,’’ one for spin up and one for spin down, that are present in the dot due to quantum interference. Because the pumped current results from the motion of the electron speckle in response to shape changes of the dot, independent speckle

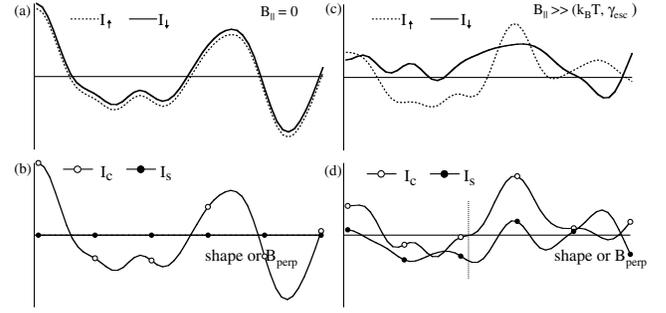


FIG. 2. Schematic plots of the pumped current dependence with an external parameter, such as dot deformation or perpendicular magnetic field. (a),(b) No in-plane magnetic field B_\parallel is applied. The spin up I_\uparrow and spin down I_\downarrow components of the charge current are equal and no spin current I_s is present. (c),(d) When a sufficiently strong in-plane field is applied, the spin component of the charge current responds in distinct ways to an external perturbation. As a result, total charge and total spin currents in the dot become uncorrelated. The vertical dashed line in (d) indicates a point where only spin is transferred across the dot with no net charge transport.

patterns are all that is needed to produce spin pumping. It is not necessary to significantly polarize the dot whatsoever.

Let us call $Q_{\uparrow,\downarrow}$ the spin up/down charge transferred after the completion of one cycle,

$$Q_{\uparrow,\downarrow} = \int_0^{2\pi/\omega} dt I_{\uparrow,\downarrow}(t). \quad (2)$$

For a chaotic or disordered quantum dot connected to leads with many propagating channels ($N \gg 1$), the variance of pumped charge over an ensemble of equivalent dots (e.g., differing in shape or disorder configuration) has been calculated by several authors [4,7,9]. We generalize these calculations, as presented in Ref. [9], to include a Zeeman field [16]. For our purposes, it will be sufficient to consider the theory in the limit of high temperature, when $\hbar\omega \ll E_Z, k_B T, \hbar\gamma_{\text{esc}}$ [17]. The resulting analytical expression for $Q_{\uparrow,\downarrow}$ is further simplified if we restrict our analysis to the case of small external oscillating voltages. This allows us to use an expansion in powers of A_1 and A_2 and retain only the leading bilinear term. Following Ref. [9], we obtain

$$\overline{Q_\uparrow Q_\downarrow} = \frac{16\pi e^2 g C_1 C_2 \sin^2 \phi}{N\Delta} \int_0^\infty d\tau e^{-N\tau\Delta/\pi} (1 + N\tau\Delta/\pi) [F(\tau)]^2 \cos(E_Z \tau), \quad (3)$$

where $g = N_r N_l / N$, $\phi = \phi_1 - \phi_2$, $F(\tau) = T\tau / \sinh(2\pi T\tau)$ (we take $\hbar = k_B = 1$ hereafter). The factors $C_{1,2}$ are related to the quantum dot response to shape deformations and can be determined through their relation to the quantum dot energy level susceptibility [7,9].

When the Zeeman energy is set equal to zero, Eq. (3) coincides with a similar expression in Ref. [9] for Q^2 and spinless electrons. Since $N \gg 1$, the exponential factor dominates the integrand decay in Eq. (3) at low

temperatures. In that case, the variance of total spin transferred per cycle, $Q_s = Q_\uparrow - Q_\downarrow$, will depend strongly on N .

The integral over τ can be evaluated numerically, yielding results such as those shown in Fig. 3, where we have plotted the quantity $r_{\text{pol}} = Q_s^2/Q_c^2$ versus E_z for several values of T and N , with $Q_c = Q_\uparrow + Q_\downarrow$ and $\phi \neq 0, \pi$. Notice that at $E_z = 0$, $Q_\uparrow Q_\downarrow = Q_\downarrow Q_\uparrow$, thus $r_{\text{pol}} = 0$. As E_z grows, the amounts of up and down spin transferred per cycle become uncorrelated. The typical amplitude of spin transfer depends strongly on temperature. The dependence on N , which is pronounced at low temperatures, decreases substantially when T is of order Δ [see Fig. 3(b)].

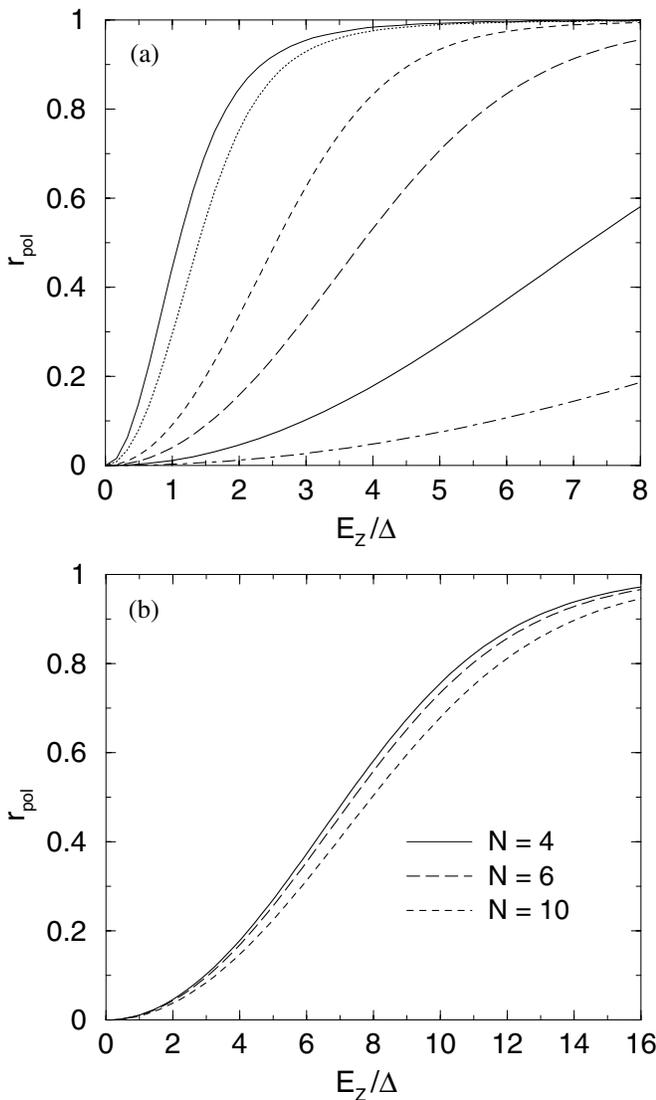


FIG. 3. Relative spin polarization of the pumped current as a function of Zeeman energy for: (a) $N = 4$ and different temperatures: $T/\Delta = 0$ (upper curve), 0.1, 0.3, 0.5, 1.0, and 2.0; (b) $T = \Delta$ and different numbers of channel.

From Eq. (3) we can estimate the typical Zeeman energy E_z^* necessary to achieve $r_{\text{pol}} = 1/2$, i.e., that spin polarize $\sqrt{1/2} \approx 70\%$ of the pumped current. When $T \ll 2\pi\gamma_{\text{esc}}$, we obtain $E_z^* \approx 1.17 \gamma_{\text{esc}}$, while in the opposite limit, $E_z^* \approx 1.49 T$. For a GaAs quantum dot with $1 \mu\text{m}^2$ in area at 100 mK and 2 T, we find that the pumped current is typically 60% spin polarized ($r_{\text{pol}} = 0.36$) when the total number of propagating channels in the leads is four. We remark that Eq. (3) indicates that the spin pumping strength should be maximal for the smallest number of propagating channels possible, namely, two.

Spin-flip scattering limits the efficiency of the spin current pump. While several mechanisms could cause spin flipping, perhaps the most relevant one to semiconductor materials is spin-orbit coupling caused by asymmetries in the confining potential and lattice structure. In a small quantum dot at $B_{\parallel} = 0$, there is a substantial reduction of the spin-orbit scattering rate as compared to the bulk two-dimensional electron gas in a GaAs heterostructure [18,19]. However, it is also known that the presence of an in-plane magnetic field (such as the one needed for the operation of the spin pump) alters significantly weak localization corrections of the conductance in laterally confined quantum dots [14,19–22], suggesting an enhancement of spin-orbit effects at $B_{\parallel} > 0$. This enhancement does depend strongly on the size of the quantum dot, as observed experimentally by Folk *et al.* [14] and theoretically examined by Halperin *et al.* [19]. For example, for the dots in Ref. [14], there is a crossover to strong spin-orbit coupling for large dots ($8 \mu\text{m}^2$ in area), while no substantial spin-orbit effects are detected for smaller dots ($1 \mu\text{m}^2$ in area). These results suggest that for small quantum dots, in the regime of temperatures and Zeeman energies that we discussed above in our estimation for $1 \mu\text{m}^2$ dots, spin-orbit scattering should not be sufficient to destroy the spin pumping mechanism we propose.

Another relevant question to be considered is whether the dc current spin polarization effect caused by pumping in the presence of an in-plane Zeeman field could be also generated by a rectification mechanism [10]. The answer is positive, since spin polarization also appears when there is a difference between the quantum dot charge conductance for up and down spin channels. That is, provided $G_\uparrow(t)$ and $G_\downarrow(t)$ oscillate with distinct amplitudes, for a voltage drop $V(t)$ we would have $\overline{I_\uparrow(t)} \neq \overline{I_\downarrow(t)}$, where $\overline{I_{\uparrow,\downarrow}(t)} = \overline{G_{\uparrow,\downarrow}(t)} V(t)$ (here the overline denotes time average). Notice, however, that while rectification would make $I_s(B_{\text{perp}}) = I_s(-B_{\text{perp}})$, a quantum pumped spin current does not need to satisfy this symmetry requirement. Thus, when both mechanisms are present, the quantum pumping component can be partially separated by extracting the symmetric part of $I_s(B)$. Another distinct feature of pumping is that it causes spin transfer without voltage drop.

Recently, it was suggested [23,24] that while parametric pumping does not survive the loss of phase coherence, another mechanism of charge transfer comes into play when dephasing is strong. We believe, however, that this incoherent mechanism cannot be used to produce a spin pumping. The reasoning is as follows. Charge dephasing affects both quantum pumping and rectification mechanisms for generating dc spin-polarized currents. In both cases, dephasing washes out the intricate wave function interference patterns responsible for fluctuations in the conductance. Even if the dephasing rate $\tau_\phi^{-1} < E_Z$, the wave function content of spin up and spin down transport matrix elements will become essentially the same. In that case, we expect $I_\uparrow \approx I_\downarrow$ and therefore no net spin current.

Finally, we emphasize that, provided the quantum dot is maintained open during the pumping cycle, the Coulomb interaction does not alter the predictions of random matrix theory upon which our analytical calculations are based [25]. Nevertheless, in principle, spin pumping could be achieved in closed dots by somewhat related mechanisms [26]. In that case, Coulomb blockade should be taken into account. Electron-electron interactions are also fundamental in quantum wires [13] and may lead to effects such as spin transport quantization.

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- [17] By restricting our analysis to this limit, we avoid having to consider heating and nonequilibrium effects. For example, in Ref. [6], $\omega/2\pi \approx 50$ MHz ≈ 0.11 μ eV.
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