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Our group has been dealing with various aspects of theoretical quantum information theory.

1. QUANTUM RATE-DISTORTION THEORY [1,5]. The quantum lossless source coding theorem specifies the minimum rate, called the entropy and measured in code qubits per source qubit, to which a quantum source can be compressed subject to the requirement that the source qubits can be recovered perfectly from the code qubits. In realistic applications we may be able to tolerate imperfect recovery of the source qubits at the receiver, in which case we would seek to minimize the rate required to achieve a specified level of distortion. Equivalently, we may be required to use a rate less than the entropy of the source, in which case we would seek to minimize the distortion subject to this rate constraint. Here, the distortion measure is a user-defined function of the input and the reconstruction, the precise form of which depends on the nature of the application. Analysis of the tradeoff between rate and distortion is the subject matter of rate-distortion theory, and its classical counterpart is an important and fertile area of information theory. In the classical case a theorem by Shannon reduces the task to constrained minimization of mutual information between two random variables. In the quantum case no such theorem appears to exist, which renders the problem almost intractable. For the first time after the quantum formulation of the problem by Barnum we succeed in finding an exact rate-distortion function, using an important distortion measure based on entanglement fidelity.

2. FIDELITY TRADEOFF [2]. It is a well-known fact that given a single copy of a quantum system it is in general impossible to determine its quantum state exactly. Of course, the situation becomes different when one is given an ensemble of identically prepared quantum systems. With the increasing size of such an ensemble one can extract more and more precise information on its preparation. An important effect which usually accompanies an attempt to determine the unknown preparation of a quantum system is the disturbance of its original state -- a penalty for gaining classical information from a quantum system. This fundamental feature of quantum mechanics has been discussed from many different points of view, depending on the particular physical scenario considered, but the problem has hitherto been analytically intractable due to the choice of the information gain measure. A recent paper by Banaszek addresses the issue by finding the tradeoff function for the single copy case between the operation fidelity F, which denotes the average overlap between the state of the system after the operation and the original one, and the estimation fidelity G, which tells us how good an estimate one can provide on the basis of the knowledge of the classical outcome of a given operation. In the joint work [2] done with Banaszek, then a post-doc in the Rochester group, we discuss a more general case of the fidelity trade-off when one is given a finite ensemble of N identically prepared systems. Without imposing any restrictions on the generality of quantum operations considered, we manage to reduce the original O(2^N) dimensional constrained minimization problem to that of diagonalizing certain NxN tridiagonal matrices. We can solve the latter numerically for any N.

3. REMOTE STATE PREPARATION [3,5]. We all know what state preparation is: Alice has a classical description of the a state, and she makes a physical instance of it in her lab. Remote state preparation is when she again has a classical description of the the state, but now wishes to prepare it in Bob's lab, Bob being far away. We have two types of resources at our disposal: shared EPR pairs (ebits) between Alice and Bob and classical bits of forward communication from Alice to Bob. This problem may be defined with or without allowing classical back-communication from Bob to Alice. In a recent preprint Bennett et al.(Phys. Rev. Lett. 87, 077902) ask about the optimal tradeoff curve in the(bits,ebits) plane necessary for Bob to be able to produce an asymptotically perfect copy of the state, i.e. so that the average error per qubit state tends to zero as the number of states to be remotely prepared goes to infinity. One achievable point in this plane is (2,1) which corresponds to teleportation. Our work [3] concentrates on the low entanglement region where the number of ebits per state is less than one. The method used by Lo and Bennett et al. is to send partial classical information about the qubit state, thus reducing its posterior von Neumann entropy(i.e. the von Neumann entropy of the posterior density operator reflecting Bob's knowledge about the qubit state having received the classical information). This may be used to Schumacher compress a block of such qubits, and teleport them at a rate corresponding to the posterior von Neumann entropy. We put this idea on a firm coding theoretical footing and use a unique blend of classical and quantum information theoretical techniques to find an expression for the optimal tradeoff curve attainable by the above method. In addition [5] contains new upper bounds on the high-entanglement region, improving on those of Bennett et al.

4. THE UNION OF PHYSICS AND INFORMATION [4]. Coming from the perspective of quantum information theory, we have recently achieved a union of quantum mechanics, thermodynamics and Shannon's information theory. The viewpoint we take is that the totality of our conceptual experience can be described in terms of correlated random variables. Two protagonists sharing the same physical world is no more than classical correlations between the states of their knowledge regarding that world. Similarly, the observation of definite physical laws is no more than classical correlations between states of knowledge regarding two consecutive acts of measurement, or preparation and measurement, depending on the experiment. The mathematical model presented in [4] seeks a middle way between Copenhagen quantum mechanics, where the wave function does not aspire to describe reality but merely represents our knowledge of it, and Everett's many-worlds interpretation which attempts to include the observer as just another physical system, leading to the unresolved basis problem. Our model includes both physical entities *P* and states of knowledge K. The two have a mathematically equivalent Hilbert space description, but the latter are imbued with a preferred basis, the elements of which are labeled by the mutually exclusive possibilities. As in Everett's theory, measurement is described as entangling elements of K (the "observers") with those of *P* via unitary transformations. Preparation, a novel concept in Everett-like theories, is treated analogously. The joint random variable describing the correlations between the states of knowledge of the various protagonists comes about by restricting attention to the diagonal elements of their joint density matrix in the preferred basis ("diagonal vision"), and Shannon mutual information is easily calculated. This framework has already led to important contributions in quantum information theory and quantum thermodynamics. In relation to the former we have shown how both quantum and classical information processing can be seen from a unified viewpoint of entanglement transfer. Regarding the latter, we provide the long overdue quantum measure of thermodynamic entropy, having the crucial property of being able to increase in a closed system. This work will be submitted to the Young Researchers Competition in Honor of John Archibald Wheeler on "Science & Ultimate Reality".

5. FUTURE DIRECTIONS: Work in progress includes a collaboration with Konrad Banaszek (formerly a member of the MURI), and Howard Barnum on the Shannon information gain vs. operation fidelity tradeoff. Also, the thesis work on high-entanglement remote state preparation will be made into an article and submitted, most probably to IEEE IT. [1] should be extended to other distortion measures. The new unifying vision presented in [4] is yet to be fully exploited in viewing the current achievements of quantum information theory in a new light, and pointing the way to further progress. It also may have a role to play in unifying the various areas of physics into a coherent theory.

References

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