A simple experiment for discussion of quantum interference and which-way measurement

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We have developed a which-way experiment using visible light that is completely analogous to a recent experiment involving which-way measurement in atom interference. This simple experiment, easily accessible to undergraduate students and the resources of undergraduate departments, facilitates the examination of the key elements of which-way measurement, quantum erasure, and related mysteries of quantum measurement. The experiment utilizes a Mach–Zehnder interferometer, and visually demonstrates the loss of interference fringes when a which-way measurement is imposed, and the restoration of that pattern when the which-way information is destroyed. This device is also sensitive enough to observe interference fringes arising from single photons. We present a simple analysis of the interference appropriate for the coherent classical field limit and the single photon limit at a level accessible to undergraduates. We also briefly mention related issues on the nature of the photon.
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I. INTRODUCTION

The mysteries of complementarity and the measurement process in quantum mechanics are probably most simply and elegantly stated in the context of interference. A quantum wave travels from a source to a detection region by two distinctly different paths. The interference between waves traveling by the two paths is apparent in the detection region by interference fringes—a spatial dependence of the probability distribution. However, if one asks a "particle question" and interrogates the wave function *en route* to identify the path taken by the particle, the interference pattern is destroyed. In the language of complementarity, by forcing particle properties on the wave in asking the "where" question (commonly referred to as a "which-way" measurement), we lose the wave properties evident in the interference fringes.

The typical explanation for the loss of the interference fringes, attributed originally to Niels Bohr, assigns the blame to the inevitable disruption associated with the measurement process. An interrogating probe (such as a photon) must have a sufficiently short wavelength to be able to distinguish the two paths, and in the process of interacting with the portion of the wave traveling by one of the paths, introduces a recoil momentum large enough to destroy the coherence between the two parts of the wave function. But is this sizable momentum kick indeed a requirement of the measurement process, or is it a coincidence of repeated application of the same uncertainty principle? In other words, are there ways of measuring or encoding which-way information that do not introduce this momentum uncertainty?

A recent paper by Dürr, Nonn, and Rempe[1] describes an experiment with exactly this sort of "kick-free" encoding. Their technically remarkable experiment uses single-atom interference to exhibit a destruction of interference fringes when different microwave transitions are applied to the two distinct atom paths, thus encoding the which-way information in a subtle way in the spin structure of the atom. In particular, the microwave transitions provide a superposition of two spin states with different phases between the two components: even for one of the paths, and odd for the other path. The photons that introduce this encoding are too soft to destroy the interference through a momentum kick. Nonetheless, the interference fringes disappear once the which-way information is stored.

Reading this article stimulated us to consider analogous processes in much simpler experimental setups using light rather than atoms. We have undertaken a set of experiments using a Mach–Zehnder interferometer, easily accessible to most undergraduate physics departments, which illustrate the same principles. After our apparatus was designed, Schwindt, Kwiat, and Englert[2] described a conceptually similar (although much more elaborate) apparatus to quantify the degree of which-way information stored. Several other authors have proposed similar experiments.[3]

In our experiment which-way information is encoded by plane polarization. With easily available technology, our apparatus can also see the interference fringes well below the average occupancy of one photon in the interferometer at a time, implying that the interference process is one photon interfering with itself, and not some more complicated interaction among many photons. In complete agreement with the other experiments, the interference pattern disappears when the which-way information is encoded. In contrast to the atom experiment, we also describe an eraser: a method of destroying the which-way information after the photon has left the interferometer, thereby restoring the interference pattern. The existence of this eraser suggests that the interference information, while not apparent in a fringe pattern, must really still be present in the photon state. We describe how this interference information is stored, and suggest some implications this form of storage might have for atom experiments such as that of Dürr, Nonn, and Rempe.[1]

II. THE INTERFEROMETER

The design of our interferometer is a modified Mach–Zehnder interferometer, shown in Fig. 1. This type of an interferometer is conceptually simple and is analogous to the two-slit experiment in many ways. A beam splitter divides the photon beam into two paths which are then redirected with mirrors and recombined at a second beam splitter. (For simplicity, Fig. 1 omits the second recombined beam that

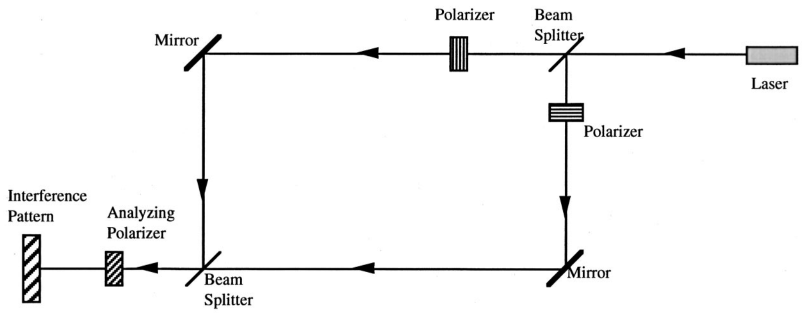

Fig. 1. Schematic view of the interferometer.

emerges downward in the figure from the second beam splitter.) The recombined beam shows interference fringes that result from geometrical path differences between the two legs; the adjustment of the intervening mirrors can change the character of these fringes from closely to widely spaced, as well as the orientation of the fringes.

Our interferometer is modified to include linear polarizers in each leg of the interferometer. This modification allows us to encode the path of the photon in the polarization of the photon. One leg is given a horizontal polarization, the other leg a vertical polarization. If one considers the angular momentum eigenstates of the photon to be a logical set of basis states, then the linear polarization states are simply even and odd combinations of those basis states, in complete analogy to the atom experiment in Ref. 1.

One could also use a polarizing beam splitter as the first beam splitter as in Ref. 2; we chose not to do so, allowing us more control of the polarization state of each leg. It also allowed us to use a more unusual beam splitter that is in closer analogy with the double-slit experiment. For most of our measurements, in place of a conventional beam splitter, we used a first surface mirror protruding partway into the light path, cutting off only half of the photon beam. This configuration then makes a left–right selection that is much akin to the double-slit case. It also sidesteps some ambiguity in the nature of the beam splitter, which is simple in a classical wave sense, but more mysterious as a device that can separate a single photon into two spatially separated pieces.

After the beam leaves the interferometer, we analyze the recombined beam with linear and circular polarizers. Classically, this analyzing polarizer allows us to examine the spatial dependence of the light polarization; quantum mechanically, it either selects a particular beam path or erases the which-way encoding in the photon's quantum state.

Finally, we attenuate the laser beam before the first beam splitter by approximately nine orders of magnitude using a combination of neutral density filters and crossed polarizers. We considered as a figure of merit the average photon occupancy, defined as the incident photon rate in photons per second times the transit time (length divided by the speed of light) of the interferometer. For our nine-orders-of-magnitude attenuation, the average photon occupancy in the interferometer is approximately 10^{-2} photons. Using an inexpensive consumer grade charge coupled device (CCD) camera, we are still able to recover clear evidence of interference fringes.

III. RESULTS

If the analyzing polarizer is oriented in the x direction, we are only looking at light that has traveled one leg of the interferometer, and no interference fringes are observed. The analyzer oriented in the y direction selects the other leg, and again no fringes result. Somewhat more interesting is the lack of interference fringes even if we have no final analyzer. It is not necessary to actually detect the particular path; just the encoding of the which-way information is sufficient to destroy the fringes.

However, this destruction of the interference fringes is *not* the result of random momentum kicks destroying coherence. We can demonstrate this by erasing the which-way information and regaining the fringe pattern. The which-way encoding is erased by orienting the analyzing polarizer at a 45° angle and thereby restoring the fringe pattern. In addition, when the final analyzer is placed at a $-45°$ angle, we also observe fringes, but in this case the previously dark areas of the fringe pattern become the light regions. Figures 2(a)–2(c) show the recombined beam without analyzer, with analyzer at a 45° angle, and with analyzer at a $-45°$ angle, with a cross mark on the projection screen to indicate the registration of the patterns.

The interference pattern seen with the light intensity reduced to sub-single-photon average occupancy requires that the interference pattern be focused onto a relatively small region of the CCD camera, illuminating something of the order of 100 pixels. The images of the interference pattern are easily seen in real time without any signal integration or adjustment of CCD readout rate. Sample images are seen in Figs. 3(a) and 3(b), where Fig. 3(a) includes the which-way encoding and Fig. 3(b) does not. The interference pattern was adjusted to be particularly broad, with a single dark band through the middle, to make the interference most apparent in the still photograph. When viewed live, the fringes are

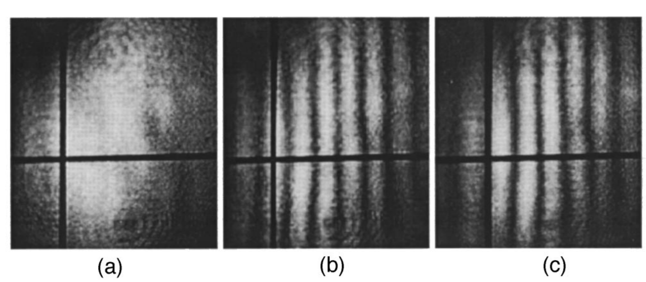

Fig. 2. Photographs of the interference patterns projected on a white screen. (a) The pattern resulting when which-way information is encoded in the polarization; (b) the pattern given after a final analysis by a 45° polarizer; (c) analyzed with a $-45°$ polarizer. The black cross is only to allow comparison of registration between figures. Note that (b) and (c) are complementary.

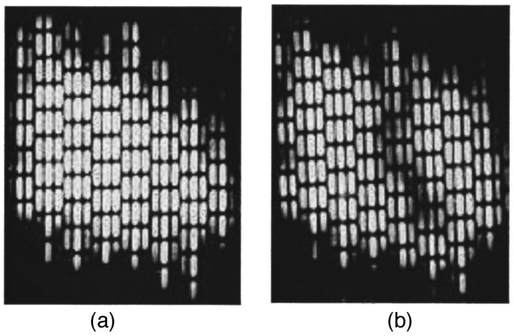

Fig. 3. (a) Photograph of a live CCD video image of a low-intensity interference pattern with which-way information encoded. (b) The same configuration but with a 45° analyzer after the polarimeter; note the single dark fringe through the center of the pattern.

quite obvious if one places a small amount of stress on one of the mirror mounts and causes the fringe pattern to shift.

IV. THEORETICAL ANALYSIS

A theoretical analysis that spans the range from coherent electric fields to single-photon limits relies on quantized electric and magnetic field operators and photon annihilation and creation operators. Such an analysis has been presented previously in this journal[4] and elsewhere[5] and will not be repeated in this article. Instead, we present a classical analysis and a simple quantum analysis accessible to a typical undergraduate physics major.

A. Classical field analysis

We can analyze the interference pattern easily for a classical electric field. After recombination by the second beam splitter, we have two superposed electromagnetic plane waves with different polarizations, and inevitably, some slight misalignment that is responsible for the visible fringe spacing. We can write this sum as

$$\mathbf{E}_{net} \approx E_0 e^{-i\omega t}[\hat{x} e^{i(k_x x + k_z z)} + \hat{y} e^{i(-k_x x + k_z z)}]$$
$$= E_0 e^{i(k_z z - \omega t)}[\hat{x} e^{ik_x x} + \hat{y} e^{-ik_x x}], \quad (1)$$

where we have assumed that the two plane waves are nearly directed in the z direction, but symmetrically oriented giving rise to equal but opposite x components of the propagation vector $\mathbf{k}$. We have also neglected the small z components of the electric field resulting from this misalignment of the propagation vectors relative to the z direction, which is why Eq. (1) is written as only approximate. The intensity of this wave is proportional to the square of the electric field, which has a spatial dependence at the screen (that is, at some constant z value) which is given by the absolute square of the final bracketed expression in Eq. (1), which simplifies to

$$E_x^2 + E_y^2 \propto E_0^2 |e^{ik_x x}|^2 + E_0^2 |e^{-ik_x x}|^2 = 2E_0^2, \quad (2)$$

that is, a constant with no visible fringes. However, if we use a polarizer to take the component of $\mathbf{E}$ along a 45° angle between $\hat{x}$ and $\hat{y}$, we find the analyzed electric field is

$$\mathbf{E}_{\text{analyzed}} \approx E_0 e^{-i\omega t}[\hat{d} e^{i(k_x x + k_z z)} + \hat{d} e^{i(-k_x x + k_z z)}]$$
$$= 2E_0 \hat{d} e^{i(k_z z - \omega t)} \cos(k_x x), \quad (3)$$

where $\hat{d}$ is the unit vector along the 45° diagonal. The intensity now shows fringes with a $\cos^2(k_x x)$ spatial dependence. In a sense, this interference was contained in the electric field expression (1), although stored in the polarization information, rather than in the intensity.

B. Quantum analysis and interpretation

This experiment serves as a simple, effective, and inexpensive vehicle for demonstrating and fostering discussion of issues of complementarity and quantum measurement. The notion of a quantum eraser—a measurement that destroys which-way information—is embodied in the postinterferometer 45° polarizer. The existence of this eraser gives insight into the which-way encoding; for instance, Bohr's argument that the interference pattern is lost because the photon direction has been modified in a random way in the measurement process is clearly inapplicable.[6] Dürr, Nonn, and Rempe[1] refuted that argument in the case of their atom interference experiment by a detailed calculation of the recoil momentum associated with microwave photon absorption. One need not go through such a calculation here because the eraser restores the interference pattern; clearly there could not have been any such random disruptive process or one would not be able to retrieve the interference pattern.

The experiment also provides a stimulus for discussion of the quantum measurement process. Although it is difficult to define what is necessary to cause a measurement, we can empirically define when a nontrivial measurement has happened if the resulting state has become an eigenstate of the measurement variable. In other words, if we follow a first, nontrivial measurement (nontrivial meaning it can yield more than one possible result) with a second measurement which reproduces the original measured value with 100% certainty, then a measurement must have occurred. Issues relating to quantum measurement and polarizers continue to be topics of discussion and interpretation.[7]

In the interferometer case, however, we produce a more subtle change in the photon state. We can represent this symbolically through a total state vector that has both spatial and spin degrees of freedom. In general, any which-way marking in an interferometer contains these two degrees of freedom, which we will denote as R defining the path route and M describing the internal change, either within the interfering particle itself or some other object, made to mark the path chosen. The which-way marking then produces a correlated state that looks schematically like[8]

$$\frac{|R_1, M_1\rangle + |R_2, M_2\rangle}{\sqrt{2}}. \quad (4)$$

In our interferometer case, the R degree of freedom specifies which leg of the interferometer, and M specifies the polarization, either x or y, of the photon.

If we were dealing with a nonrelativistic particle such as an electron, neutron, or atom, we could then proceed to actually specify a wave function that is a product of a conventional wave function and the spin degree of freedom. However, a proper treatment of the photon demands a field theoretic approach,[9] and one cannot describe the photon in generality by a wave function.[10] However, we borrow a technique from Baym[11] in which we only look at the polarization degree of freedom, and use that to infer the appropriate leg of the interferometer. Our orthonormal basis states will then be simply x and y polarizations, which we will denote as $|x\rangle$ and $|y\rangle$. We will assume that we begin with a 45° polarized beam (as in fact we do in our experiment, the laser being polarized), and therefore we can use a superposition of states and not have to use the density matrix to describe an arbitrary polarization, including a possible unpolarized component in the form of a mixture.[5,11] Before entering the polarimeter, the photon polarization state is

$$|\Psi_{\text{before}}\rangle = \frac{|x\rangle + |y\rangle}{\sqrt{2}}. \tag{5}$$

A polarizing beam splitter separates these two components into the different legs, and then they are recombined in the final beam splitter (with a 50% efficiency) giving a recombined polarization state, within an overall phase, of

$$|\Psi_{\text{after}}\rangle = \frac{|x\rangle + e^{i\varphi}|y\rangle}{2}. \tag{6}$$

In this expression, φ is a phase difference introduced by differences in the path lengths of the two legs of the interferometer, which in general will be a function of a transverse direction, say x, as was the case in the classical analysis. Because of the orthogonality of the basis states $|x\rangle$ and $|y\rangle$, the phase factor φ is not observable in the intensity; no fringes are visible.

One might complain that no interference should be expected in the final recombined beam anyway, because the state of the photon is simply different depending on which path it traveled; because the final states are distinguishable, they cannot interfere. However, we have already shown experimentally that such interference is indeed recoverable with an analyzing polarizer. We can also show analytically that an interference of a sort still exists if we look for it in the polarization component of the final state. If we look for the expectation value of the polarization at 45°, we can see an observable effect. Let us define a polarization operator in the 45° direction by its effect on the basis states, that is,

$$\hat{P}_{45}|x\rangle = \frac{|x\rangle + |y\rangle}{2}, \quad \hat{P}_{45}|y\rangle = \frac{|x\rangle + |y\rangle}{2}, \tag{7}$$

which then gives us an expectation value for this operator after the beam recombination

$$\langle\Psi_{\text{after}}|\hat{P}_{45}|\Psi_{\text{after}}\rangle = \langle\Psi_{\text{after}}|\frac{(|x\rangle + |y\rangle) + e^{i\varphi}(|x\rangle + |y\rangle)}{4}$$

$$= \frac{1+\cos(\varphi)}{2}, \tag{8}$$

which is then revealed as interference fringes after the analyzer.

It is also satisfying to look at analysis by a polarizer at $-45°$. The corresponding operator can be defined as

$$\hat{P}_{-45}|x\rangle = \frac{|x\rangle - |y\rangle}{2}, \quad \hat{P}_{-45}|y\rangle = \frac{-|x\rangle + |y\rangle}{2}, \tag{9}$$

which gives the expectation value

$$\langle\Psi_{\text{after}}|\hat{P}_{-45}|\Psi_{\text{after}}\rangle = \frac{1-\cos(\varphi)}{2}, \tag{10}$$

which is complementary to the previous interference pattern.

We can gain some insight into the restoration of the interference by looking at the expression for the recombined photon state in Eq. (6). As we increase the phase φ between the basis states $|x\rangle$ and $|y\rangle$ from zero to 2π, the polarization varies from 45° to right circularly polarized to $-45°$ to left circularly polarized and back to 45° with elliptical states in between. Even before the final analyzer, the interference fringes have not been expunged, but reveal themselves in a spatially dependent polarization of the light. This dependence clearly agrees with our measurements with an analyzing polarizer as shown in Fig. 2; although not shown, we also were able to observe the intervening phases of circular polarization with a circular polarizer.

V. ENTANGLED STATES

The analysis of this simple interferometer case also suggests an interpretation of the loss of interference in more complicated cases such as that of the atom interference of Ref. 1. That is, the interference is not really lost, but has its expression in the spatial dependence of the spin states of the recombined atom beams.[12] One could in principle restore the spatial intensity variation through a spin analyzer analogous to our analyzing polarizer, if such an analyzer could be devised. Perhaps more interesting, the atom interferometer or variants could be used to produce exotic states of atoms having an unusual spatial dependence to the spin structure similar to that in Eq. (6).

We would be remiss to overlook the fact that the entanglement of the spatial and polarization components of the photon state in Eq. (4) is formally identical to the entanglement of two correlated spins in the classic Einstein, Podolsky, Rosen (EPR) gedanken experiment.[13] (This comparison to EPR has been made by others as well.[8,14]) For example, if a source emitted two electrons with total spin zero, the resulting spin portion of the wave function would be expressed as

$$\chi \equiv |\text{spin 1,spin 2}\rangle = \frac{|\text{up,down}\rangle + |\text{down,up}\rangle}{\sqrt{2}}, \tag{11}$$

which is formally the same as Eq. (2), with the substitution of the R spatial choice for one of the spin degrees of freedom. In both cases, we exchange what we would consider a measurement on a single degree of freedom (for example, spin of a single isolated electron) for an entanglement between degrees of freedom (for example, spin of electron 1 and spin of electron 2). As a result, all possible values of any single measurement (for example, spin of electron 1 in the EPR case or polarization in the interferometer case) are retained in the wave function, and it is only the correlations in dual measurements (for example, spin 1 *and* spin 2 for EPR, or polarization *and* position for the interferometer) that are constrained. It is interesting to note that this effect agrees perfectly with classical predictions for the interferometer case, but does not agree with classical predictions for the EPR case. This mysterious entanglement, in the language of complementarity, seems to belong in the wave character of quantum mechanics, and not in the particle nature revealed in measurements, because the classical picture of the interferometer is a wave picture, and the classical picture of the EPR case is a particle picture.

VI. NATURE OF THE PHOTON

This experiment also provides a context for discussion of a number of issues relating to the nature of the photon. We made no effort to demonstrate directly the corpuscular nature of light, but assumed the existence of the photon and determined the appropriate laser power level based on extrapolations of light intensity. Use of photon counters such as avalanche photodiodes or photomultiplier tubes would add the dimension of verifying photons, although adding expense and sacrificing the visual effect of the CCD camera image.

The tendency of photons, as bosons, to arrive coincidentally clouds the achievement of the single photon level somewhat, and draws our attention to the photon statistics of various light sources. These issues are summarized nicely by Loudon.[15] Furthermore, if the issue of one photon interfering with another is of concern, one really should take care to consider what one might consider the typical overlap of two photons, given by comparing the effective spacing and the coherence length of the photons. For our diode laser source and tabletop experiment, the coherence length of the photons is approximately an order of magnitude larger than the apparatus. Because our weakest beam had an average photon occupancy of about one hundredth, the spacing between photons was still an order of magnitude larger, on average, than the coherence length.

One can also raise issues of how photons interfere. This experiment clearly demonstrates that a single photon will produce interference in an interferometer. This would give credence to Dirac's famous dictum "Each photon then interferes only with itself."[16] However, other experiments show clear interference between two independent lasers, suggesting otherwise.[17] The notion of different photons interfering is satisfying only until one confronts similar experiments that continue to show this interference—even at the individual photon counting level![18] This surprising effect is explained by noting that one cannot tell which source the photon came from, therefore the two processes interfere; in a sense, each photon is emitted by both lasers. Most will find this explanation a little discomforting, and animated exchanges still occur in professional journals on the correct interpretation, or at least verbal description, of these effects.[19]

Finally, it is often tempting to beginning students of quantum mechanics to identify the electric field with a photon wave function. We find light intensity (and therefore photon intensity) scales with the square of the electric field, just as we expect probability to be proportional to the square of a nonrelativistic wave function. However, the coherent state that we associate with classical electric fields arises from many photons, in fact, from a state with an indeterminate number of photons.[20] One is forced to superpose an infinite number of states with different photon occupation numbers to achieve a coherent field, making it impossible to associate an electric field with an individual photon.

VII. CONCLUSION

We have described a simple experiment within the means of most undergraduate physics departments that illustrates many of the principles of current research in the foundations of quantum mechanics. It directly demonstrates interference with which-way and quantum eraser effects. It can be simply run at the single photon level without sacrificing the visible interference pattern. Simple theoretical analysis is also easily accessible to undergraduates familiar with the rudiments of quantum and electromagnetic theory.

The field of quantum measurement is a busy one, and while we will not attempt to review the literature, we wish to make a few references to related work. An article with a significant pedagogical purpose, although without experimental work, has been published by Englert.[8] The interested reader should also be aware of a long history of neutron interference experiments with spin interrogated paths,[12,21] as well as other experiments suitable for the teaching laboratory on related issues.[22] For those wishing for more resources, the articles by Ben-Aryeh, Ludwin, and Mann[5] and Englert[8] in particular have extensive reference lists.

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APPENDIX: PROBLEMS AND DISCUSSION TOPICS

(1) Argue that a polarizer qualifies as a measuring device by our empirical definition of measurement.

(2) Fill in the steps to Eq. (10) using state (6) and the operator definitions in Eq. (9).

(3) A correlation between polarization and transverse position as produced by the interferometer can also be produced without an interference process using a polarizer and a retarding material such as that used for quarter- and half-wave plates. Describe a configuration of polarizer and retarder that would create such a correlation, and describe the classical electric field that results in a form like Eq. (1).

(4) A device such as described in Problem 3 can be used to "undo" the transverse dependence of the polarization of the beam leaving the polarimeter. What effect does such a device have on the quantum measurement aspects of the beam? Can it serve as an eraser? Recall there are two recombined beams, perpendicular to one another, emerging from the second beam splitter. Does such a device placed after the first beam emerging from the interferometer have any effect on the second recombined beam emerging from the beam splitter?

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