

UNIVERSITY OF ROCHESTER  
THE INSTITUTE OF OPTICS

OPT 253, OPT 453, PHY 434

# **Lab. 1. Entanglement and Bell's Inequalities**

(see also PowerPoint slides of the lectures and other materials on  
the website of this course:

<http://www.optics.rochester.edu/workgroups/lukishova/QuantumOpticsLab> )

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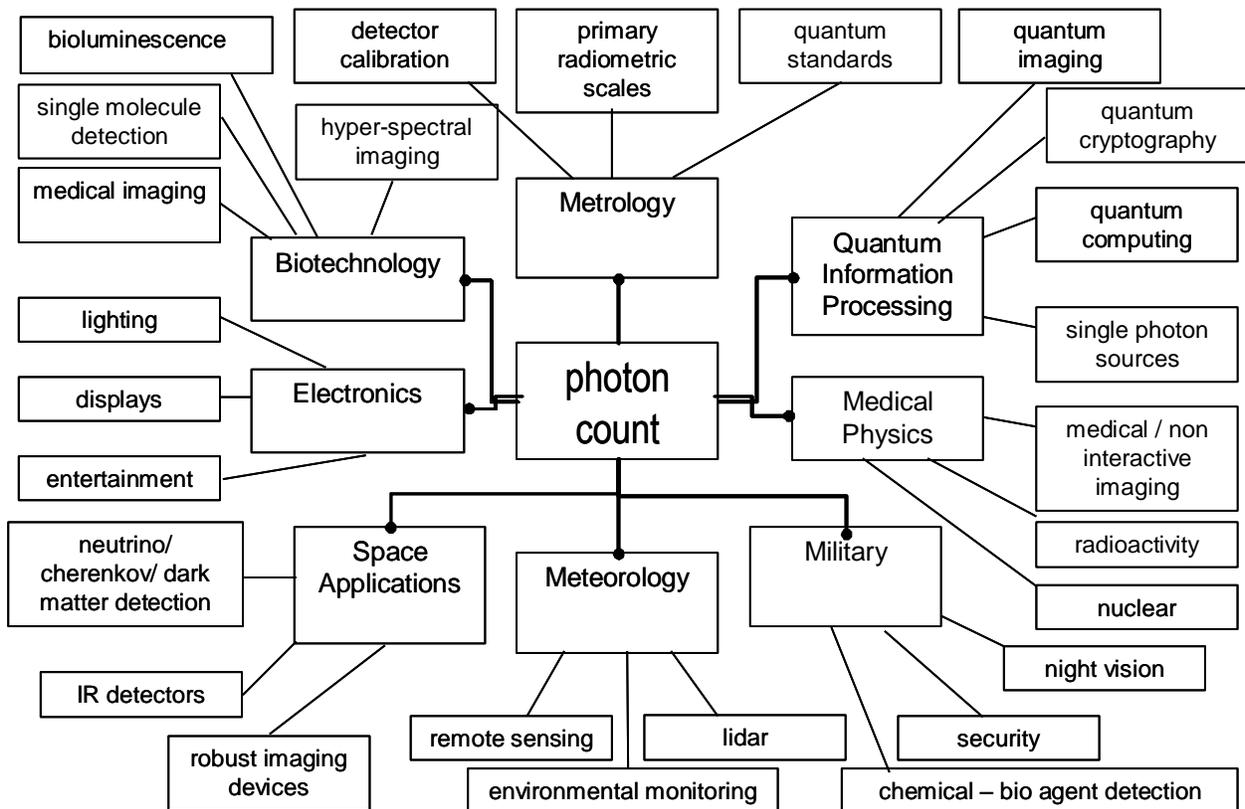
Fall 2018

*"I cannot seriously believe in quantum theory because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance."* - Albert Einstein

## Summary of this Lab

- Entanglement is the most exciting and mysterious property of some quantum mechanical systems when property of one particle depends on the property of the other. It does not matter how far apart such entangled particles are located. Among the best known applications of entanglement are quantum communication, quantum computing and quantum state teleportation.
- Bell's inequality is a classical relation. For entangled particles it is violated.
- In this lab you will obtain an entangled state of two photons and will calculate Bell's inequality using measurements of the coincidence counts between two single-photon detectors. You will work on modern, cutting edge photon counting instrumentation widely used in quantum information science and engineering. The table below shows its other applications.

## Photon counting applications



Areas of applications of photon counting instrumentation (prepared by organizers of second international workshop "Single Photon: Sources, Detectors, Applications and Measurements Methods" (Teddington, UK, 24-26 October 2005)).

# IMPORTANT SAFETY TIPS

## A. LASER SAFETY

The blue diode laser from Toptica Photonics (405.5 nm wavelength) emits laser radiation that can permanently damage eyes and at its full power (120 mW) it can damage skin as well. **To minimize the risk of injury or expensive repairs, carefully follow these instructions.**

### PRECAUTIONS FOR THE SAFE OPERATION OF the LASER

- You will work with 25 mW power (from maximum 120 mW). In this case you will not use a protective eyewear. If you need alignment of a set up, you can further reduce the power of the laser.
- Avoid looking directly to the output beam at any power level, even diffuse reflections can be hazardous.
- Try to maintain a high ambient light level in the laser operation area at high power. This keeps the eye's pupil constricted, thus reducing the possibility of eye damage.

## B. EQUIPMENT SAFETY

For the safety of the equipment,

- **NEVER TURN ON THE ROOM LIGHTS while the Single-Photon counting avalanche photodetector modules (APDs) and Electron Multiplying Charge Coupled Device (EM-CCD) camera ARE ON.**
- (1) After turning off the power supply switch always unplug the APD's before turning on the lights for the long period of time. (2) Also, make sure your Lab View program is turned off before turning on the lights.
- **If APD count rate will exceed 200,000 counts/sec, reduce laser power or put the screen in front of APD.**  
**DON'T SWITCH OFF APD UNDER A HIGH COUNT RATE!!!! APD cooling system will also be turned off and APD can be destroyed by overheating.**

# **PREPARATORY QUESTIONS (in addition to a multiple choice Quiz)**

In this Manual you will have two sets of questions:

- (1) before your first laboratory session and
- (2) after each section of this Manual. All questions have a blue-color font.
- (3) Most of these questions will be included to “At Home” and MidTerm Quizzes.

## **Try to answer these questions for yourself before your first laboratory session**

1. What is entanglement? How will you create polarization entangled photons in this experiment?
2. How will you prove in your experiment that you have entangled photons?
3. What is spontaneous parametric down conversion?
4. Make a sketch of your experimental setup and describe its main components.
5. For what purpose will you use a quartz plate in this lab?
6. What are single photon counting avalanche photodiode modules? How to work with them without damaging these detectors?
7. What are Bell’s inequalities? Can you calculate them for some classical objects?

## References and recommended literature:

1. M. Fox, *Quantum Optics: An Introduction*, Oxford University Press, 2006.
2. A. I. M. Rae, *Quantum Mechanics*, IoP Publ., 2002.
3. G. Greenstein and A.G. Zajonc, *The Quantum Challenge*, Jones and Bartlett Publ., Boston, 2006.
4. P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, "New high-intensity source of polarization-entangled photon pairs", *Phys. Rev. Lett.*, **75**, 4337 (1995).
5. D.N. Klyshko, *Photons and Nonlinear Optics*, New York, Gordon and Breach, 1988.
6. P.G. Kwiat, E. Waks, A.G. White, I. Appelbaum, P.H. Eberhard, "Ultrabright source of polarization-entangled photons", *Phys. Rev. A*, **60**, R773 (1999).
7. D. Dehlinger and M.W. Mitchell, "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory", *Am. J. Phys.*, **70**, 903 (2002).
8. D. Dehlinger and M.W. Mitchell, "Entangled photon apparatus for the undergraduate laboratory," *Am. J. Phys.*, **70**, 898 (2002).
9. J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, 2004.
10. J. Eberly, "Bell inequalities and quantum mechanics", *Amer. J. Phys.*, **70** (3), 286, March (2002).
11. M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000, pp. 111-117.
12. N. Boeuf, D. Branning, I. Chaperot, E. Dauler, S. Guérin, G. Jaeger, A. Muller, A. Migdall, "Calculating characteristics of noncollinear phase matching in uniaxial and biaxial crystals", *Opt. Eng.* **39**(4), 1016-1024 (April 2000).  
<http://physics.nist.gov/Divisions/Div844/facilities/cprad/index.html#contents>

# 1. INTRODUCTION

The purpose of this laboratory work is to introduce students to entanglement, one of the key concepts of quantum mechanics. While the concept of entanglement defies the classical intuition, in this lab you will gain a better understanding of entangled particles and experimentally verify the predictions of quantum theory.

If two particles (A and B) are entangled, their wave functions cannot be separated. The particles cannot be represented or talked about individually. Any measurement performed on A would change the state of B (and vice-versa), no matter how far apart A and B may be. The idea is illustrated in the cartoon below. There is no classical explanation for this phenomenon.

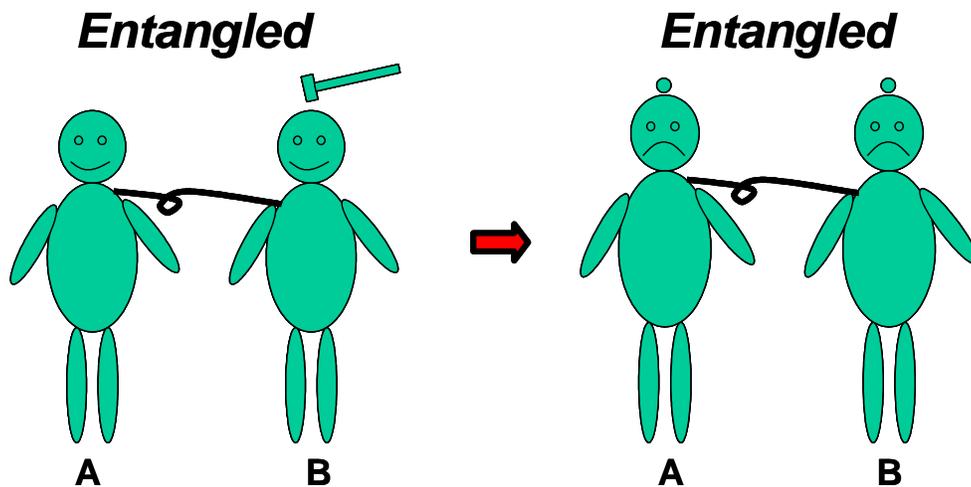


Figure 1. Cartoon of A.K. Jha and L. Elgin illustrating entanglement.

Entanglement between particles is always through some physical property. For example, the quantum mechanical state describing particles' momentum, spin or polarization may be entangled. You can read about entanglement in the books [1-3] for advanced undergraduate and graduate students.

In this laboratory two different photons whose polarization states are entangled will be investigated. These entangled photons are produced in a BBO crystal [4] through a process called **spontaneous**

**parametric down-conversion** [5]. The probability of this nonlinear optical process is very low ( $10^{-10}$ ) that is why we need to use single-photon counting detectors to record down converted photons. This lab experiment was reported in 1999 as a research paper in Ref. 6 and developed for the students' lab by the authors of Refs. 7 and 8. Ref. 7 contains also some useful historical background.

Bell's Inequality [9-11] is a mathematical equation. "Inequalities of the Bell type by themselves have nothing to do with quantum theory. Contexts as different as downhill skiers and laundered socks have been used by Meystre and Bell to demonstrate this. Although Bell inequalities are almost tautological expressions, they have attracted much attention because they allow one to see the experimental consequences of alternative views of physical reality which are conveniently labeled classical and quantum mechanics" [10].

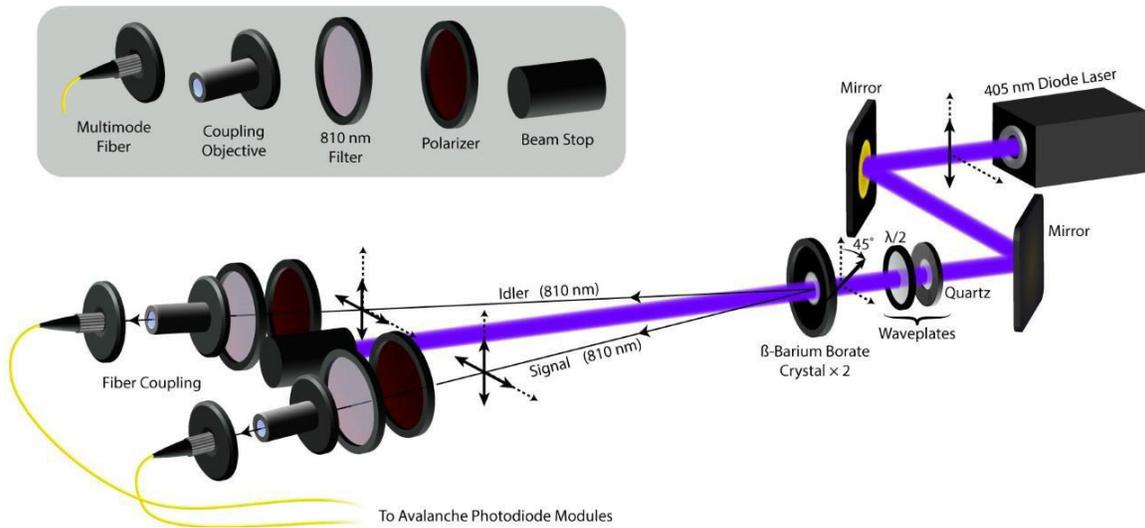
In this lab you will calculate Bell's Inequality predicting the maximum value of a sum of probabilities. The probabilities involved assume a classical correlation between two particles, whose polarization states are measured. Violating this inequality means that these particles do, in fact, have a quantum relationship that cannot be explained by classical mechanics.

## **2. EXPERIMENTAL SETUP**

As shown in the schematic diagram in Figure 2, to obtain polarization entangled photons we used light from a ~25 mW blue diode laser with a wavelength  $\lambda = 405.5$  nm and a vertical polarization. After two mirrors for alignment the laser beam passes through a half waveplate that can rotate linear polarization. Usually a quartz plate should be placed into the beam after a half waveplate, but in our experiments a quartz plate is used only in a last set of measurements. Quartz is a birefringent material. When light passes through the quartz plate, a phase difference is introduced between two polarization components. This phase difference can be adjusted by rotating the quartz plate.

The beam passes through a pair of BBO (Beta-Barium-Borate) crystals that are mounted back-to-back, at  $90^\circ$  with respect to each other. The majority of the laser light passes through the BBO crystals

(this light does not transfer to the spontaneous parametric down converted photons) and is absorbed in a beam stop. Down-converted photons are collected by a collecting system (microscope objectives and the fibers) and detected by a single photon avalanche photodiode modules (APDs). Incident light (405.5 nm) is rejected by the interference filters placed in front of a collecting system. The down-converted photons from the BBO crystals are emitted in cones [a horizontal and a vertical polarization cone for a beam with 45° incident polarization (relative to the BBO optical axis)].



**Figure 2a. Schematics of experimental setup with a blue diode laser with a 405.5 nm wavelength (prepared by R. Lopez-Rios).**



**Figure 2b. TOP: Photo of experimental setup. BOTTOM LEFT: Collection system with a 3-D adjustable mount with two polarizers and APD detectors. BOTTOM RIGHT: BBO crystal set with a 3-D rotation mounting (an iris diaphragm serves for alignment).**

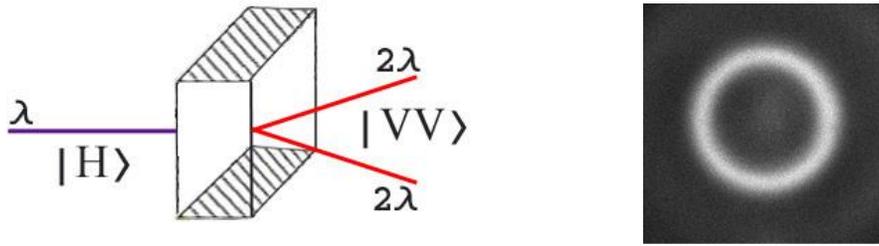
Down-converted photons from the BBO crystals with wavelength  $2\lambda = 811$  nm are detected by a pair of APDs. The x-y and z position of the collecting system objective and an optical fiber entrance relative to the objective focus can be adjusted using the x-y and z translation of the mounts. APDs are single photon detectors that give rise to an electronic signal TTL (transistor-transistor logic) pulse when a photon is incident on them.

The entrance' positions of two collecting systems are equidistant from the center of the BBO crystal set. This enables these two collecting systems to be on two diametrically opposite points of the downconverted cone. Data from the APDs are collected using a Lab View interface on a computer with a counter-timer PCI board inside.

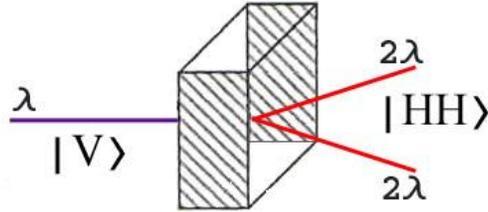
The interference filters with a 10 nm bandwidth should be placed in front of each APD, so that only light near 811 nm (i.e. the down-converted photons) will reach the detectors. **Additionally, linear plastic sheet polarizers are located in front of collecting system, so that the polarization state of photons reaching the detectors can be selected. These polarizers are very important in this lab. You will obtain photons entangled in polarization.**

### **3. BBO CRYSTALS AND PARAMETRIC DOWN-CONVERSION (see lecture materials online)**

Beta Barium Borate (BBO) is a negative uniaxial nonlinear crystal [12]. Crystals can be cut for the different types of nonlinear interactions. For a type I crystal cut, when a horizontally (vertically) polarized photon of wavelength  $\lambda$  is incident on the crystal, two photons of wavelength  $2\lambda$  emerge from the crystal with vertical (horizontal) polarization (Figure 3). This process is called spontaneous parametric down-conversion in a type I crystal and is a standard method used to produce polarization-entangled photons [4, 6]. The down-converted photons are emitted in a cone from the crystal. The efficiency of this down-conversion process is only  $10^{-10}$  (out of  $10^{10}$  incident photons only one photon would get down-converted). For details in noncolinear phase matching conditions in parametric down conversion (Figs. 3, 4) see paper 12.



**Figure 3. LEFT: Down-conversion of photon with a horizontal polarization. (For this orientation of the crystal, photons with a vertical polarization pass straight through). RIGHT – image of a cone of downconverted photons in a type I BBO crystal in a cross-section perpendicular to the incident beam. Interference filter transmits downconverted photons with a bandwidth of 10 nm.**



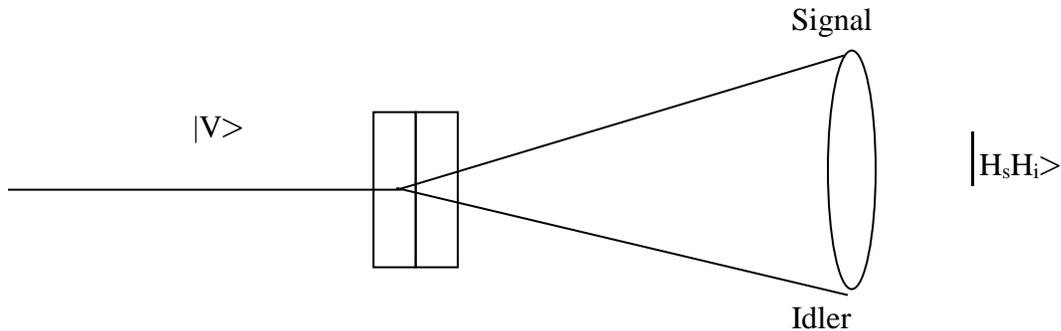
**Figure 4. Down-conversion of photon with a vertical polarization. (For this orientation of the crystal, photons with a horizontal polarization pass straight through).**

Since the majority of the laser light simply passes straight through the BBO crystals, the intensity of the down-converted photons is very low and you can not see 811 nm wavelength by your eye (although after adaptation in a darkness a human eye can see a few photons, 811 nm wavelength is outside its sensitivity range. We will use single-photon counting avalanche photodiodes (APDs) sensitive to this wavelength to observe the down-converted photons.

## **4. GENERATION OF POLARIZATION ENTANGLED PHOTONS: THEORY**

The generation of a polarization-entangled quantum state using type I BBO crystals with a 45° incident polarization is explained below. The first crystal's optical axis and the pump beam define the vertical plane. Due to Type-I phase matching, a vertically polarized photon

going through these crystals would get down-converted in the first crystal producing two horizontally polarized photons (Figure 5).



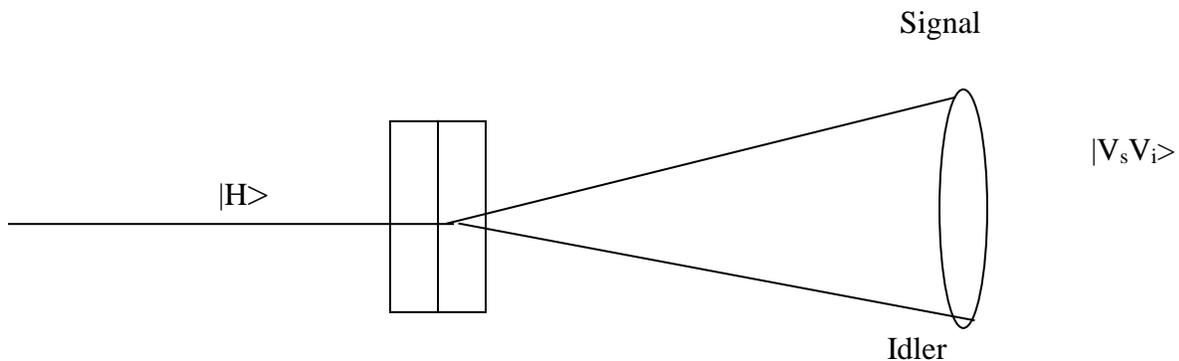
**Figure 5. Production of two horizontally polarized photons from one vertically polarized photon in a type I crystal (you can see a cone of the down-converted photons in a noncolinear parametric down-conversion with horizontal polarization).**

Mathematically it can be represented as:

$$|V\rangle \rightarrow |H_s H_i\rangle$$

Here  $V$  and  $H$  represent a horizontally- or a vertically-polarized photon. For historic reasons, photons of a down-converted pair are called “signal” and “idler” photons, denoted by subscripts “s” and “i,” respectively.

On the other hand, a horizontally-polarized photon going through these crystals would get down-converted in the second crystal producing two vertical photons (Figure 6).

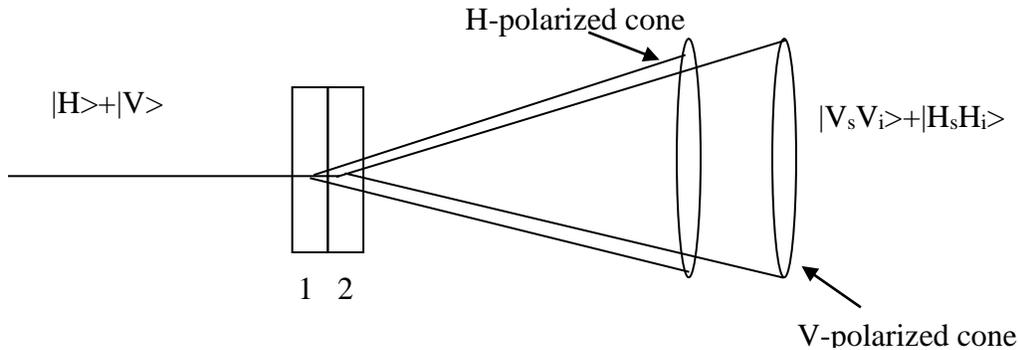


**Figure 6. Production of two vertically polarized photons from one horizontally polarized photon in a type I crystal (a cone of down-converted photons with vertical polarization).**

Mathematically it can be represented as:

$$|H\rangle \rightarrow |V_s V_i\rangle$$

Now what happens if photons with  $45^\circ$  polarization are incident on a pair of BBO crystals, as shown in Figure 7 below?



**Figure 7. Production of polarization entangled photon pairs in a type I crystal (cones with different polarizations are overlapped). The light from two crystals is not polarized (two independent sources with equal amount of photons).**

A stream of  $45^\circ$ -polarized photons can be considered as a beam with half vertically and half horizontally polarized photons. So half of the time these photons would get down-converted in the first crystal, producing pairs of photons with horizontal polarization, and half of the time they would pass through the first crystal and would get down-converted in the second crystal, producing pairs of photons with vertical polarization. Hence, for an incident beam of  $45^\circ$ -polarized photons, the same number of photon pairs having vertical and horizontal polarizations will be emitted from the two BBO crystals.

Mathematically this can be represented as:

$$\frac{|H\rangle + |V\rangle}{2} \rightarrow \frac{|V_s V_i\rangle + |H_s H_i\rangle}{\sqrt{2}} \equiv \Psi_{ent} \quad (1)$$

Here,  $\frac{|H\rangle + |V\rangle}{2}$  represents a  $45^\circ$  polarized photon.

Notice that the state  $\Psi$  cannot be factored into states purely dependent on the signal and idler photons, i.e.:  $\Psi \neq |1\rangle_s |2\rangle_i$ , for any choice of  $|1\rangle_s$  and  $|2\rangle_i$ . This means that the state of one particle cannot be specified without making reference to the other particle. **Particles related in such a manner are called entangled, and  $\Psi_{\text{ent}}$  is called an entangled quantum state.**

If we measure the polarizations of signal and idler photons in the  $H, V$  basis, there are two possible outcomes: both vertical and both horizontal. Each occurs half of the time. We could instead measure the polarizations with polarizers rotated by an angle  $\alpha$ . We use the rotated polarization basis

$$\begin{aligned} |V_\alpha\rangle &= \cos \alpha |V\rangle - \sin \alpha |H\rangle, \\ |H_\alpha\rangle &= \sin \alpha |V\rangle + \cos \alpha |H\rangle \end{aligned} \quad (2)$$

Here  $|V_\alpha\rangle$  describes a state with polarization rotated by  $\alpha$  from the vertical, while  $|H_\alpha\rangle$  describes a state with polarization rotated by  $\alpha$  from the horizontal. In this basis the entangled state is

$$\Psi_{\text{ent}} = \frac{1}{\sqrt{2}} (|V_\alpha\rangle_s |V_\alpha\rangle_i + |H_\alpha\rangle_s |H_\alpha\rangle_i) \quad (3)$$

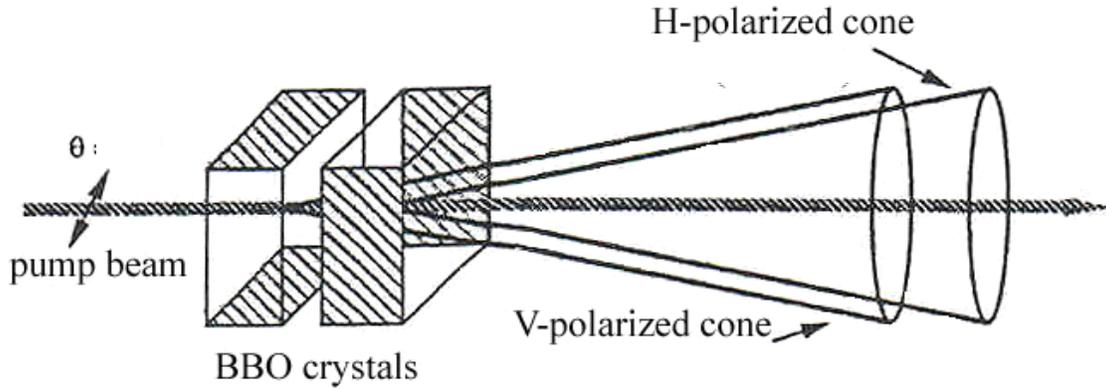
Clearly, if we measure in this rotated basis, we obtain the same results: half of the time both are  $|V_\alpha\rangle$  and half of the time both are  $|H_\alpha\rangle$ . Knowing this, we can measure the signal polarization and infer with certainty the idler polarization.

**From Figure 7 you can see that a horizontal photon travels a larger distance inside the BBO crystals than a vertical photon before getting down-converted.** This difference in distances traveled introduces a phase difference,  $\phi$ , between the two polarization states, resulting in a quantum state:

$$\Psi = |V_s V_i\rangle + e^{i\phi} |H_s H_i\rangle \quad (4)$$

If the incident (pump) laser beam has a polarization angle  $\theta$  from the vertical (see Figure 8 in which the crystal optical axis is parallel to the vertical direction), in general case the downconverted photons emerge in the state

$$|\psi_{DC}\rangle = \cos\theta |H\rangle_s |H\rangle_i + \exp[i\phi] \sin\theta |V\rangle_s |V\rangle_i \quad (5)$$



**Figure 8. Geometry of a downconversion process.**

By placing polarizers rotated to angles  $\alpha$  and  $\beta$  in the signal and idler paths, respectively, we measure the polarization of the downconverted photons.

For a pair produced in the downconverted state  $|\psi_{DC}\rangle$ , the probability of coincidence detection is

$$P_{VV}(\alpha, \beta) = |\langle V_\alpha |_s \langle V_\beta |_i | \psi_{DC} \rangle|^2, \quad (6)$$

The  $VV$  subscripts on  $P$  indicate the measurements outcome  $V_\alpha V_\beta$ , both photons vertical in the bases of their respective polarizers. More generally, for any pair of polarizer angles  $\alpha$  and  $\beta$  there are four possible outcomes,  $V_\alpha V_\beta$ ,  $H_\alpha H_\beta$ ,  $H_\alpha V_\beta$ , and  $H_\alpha H_\beta$ .

Using the basis of Eq (2), we find after some trivial calculations

$$\begin{aligned} P_{VV}(\alpha, \beta) &= (\sin^2 \alpha \sin^2 \beta \cos^2 \theta \\ &+ \cos^2 \alpha \cos^2 \beta \sin^2 \theta \\ &+ \frac{1}{4} \sin 2\alpha \sin 2\beta \sin 2\theta \cos \phi) \end{aligned} \quad (7)$$

A special case occurs when  $|\psi_{DC}\rangle = |\psi_{ent}\rangle$ , that is, when  $\theta = \pi/4$  and  $\phi = 0$ . In this case

$$P_{VV}(\alpha, \beta) = \frac{1}{2} \cos^2(\beta - \alpha). \quad (8)$$

In this lab, you will obtain a relation (8) in your experiment selecting: (1) an optimal angle  $\theta$  of crystal rotation relative to the

incident laser polarization (which is fixed) and (2) an optimal angle of rotation of a quartz plate to compensate phase  $\phi$ . **In a current setup a wavefront of a diode laser permits to obtain a relation (8) and violate Bell's inequality without a quartz plate in a setup. You will place a quartz plate at the end of this lab to see how the phase difference between two polarizations influences your results.**

In the experiment you will measure a coincidence count rate  $N(\alpha, \beta)$  choosing a fixed interval of data acquisition (1 s). Assuming a constant flux of photon pairs, the number of  $N(\alpha, \beta)$  will be

$$\begin{aligned}
 N(\alpha, \beta) = & A(\sin^2 \alpha \sin^2 \beta \cos^2 \theta \\
 & + \cos^2 \alpha \cos^2 \beta \sin^2 \theta \quad , \quad (9) \\
 & + \frac{1}{4} \sin 2\alpha \sin 2\beta \sin 2\theta \cos \phi) + C
 \end{aligned}$$

where A is the total number of entangled pairs produced, and C is an offset to account for imperfections in the polarizers and alignment of the crystals. This offset is necessary to account for the fact that some coincidences are observed even when the polarizers are set to  $\alpha = 0$ ,  $\beta = 90^\circ$ .

In an ideal case ( $C = 0$ ), if  $\theta$  is fixed as  $45^\circ$  and  $\phi$  is determined to be minimized by rotating the quartz plate:

$$\begin{aligned}
 N(\alpha, \beta) &= \frac{A}{2} (\sin^2 \alpha \sin^2 \beta + \cos^2 \alpha \cos^2 \beta + 2 \sin \alpha \cos \alpha \sin \beta \cos \beta) \\
 &= \frac{A}{2} (\cos \alpha \cos \beta + \sin \alpha \sin \beta)^2 \quad (10) \\
 &= \frac{A}{2} \cos^2(\alpha - \beta)
 \end{aligned}$$

**QUESTION 4.1.** How will a count rate of APD detector A (singles' count rate) depend on the angle  $\alpha$  of a polarizer A and on the angle  $\beta$  of a polarizer B?

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**QUESTION 4.2.** What are the conditions for maximum and minimum coincidence count rates for your setup with two polarizers in front of each APD?

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As it was described in Section 4, in this lab you will produce the quantum entangled state  $\Psi = |H_s H_i\rangle + |V_s V_i\rangle$ .

## 5. BEFORE THE EXPERIMENTS:

1. **(Caution: never switch the APD on in room light !)** ALL YOUR EXPERIMENTS SHOULD BE CARRIED OUT IN ABSOLUTE DARKNESS!!!

2. If APD count rate will exceed 200,000 counts/sec, reduce the laser power or put the screen in front of the APDs.  
**DON'T SWITCH OFF APD UNDER A HIGH COUNT RATE!**

## 6. STARTING THE LABVIEW PROGRAM

Recall that Count A and Count B in the Labview program refer to the counts in APD-A and APD-B, respectively (**singles count**). The term “**singles count**” refers to either Count A or Count B. **Coincidence count** measures the number of photon pairs simultaneously reaching the two detectors. Each APD detects singles, but using a counter-timer board within a computer, coincidence count can be measured within an accuracy of the board (26 ns in this lab).

1. Make the time window in a LabView program to be 1-5 seconds (or 1,000 – 5,000 ms). Hit the run  $\Rightarrow$  button and then “START”. Check that you have “0” counts when APDs are off. If the program is running continuously leave it like that. Otherwise, stop the LabView program by hitting . Right click on “START”. Go to ‘mechanical action’. Select ‘switch when pressed’. Hit the run

(  $\Rightarrow$  ) button and then “START”. Make sure that it is running continuously.

2. Switch the lights off. Make sure that the door is locked. Make sure that the laser is off.
3. Switch on the APDs in a presence of TA.
4. Record the counts when the laser is off and APDs are covered by the dark tissue. These are dark counts. Record the counts when the laser is on but the output from the BBO crystals is blocked by the screen. These are background counts.

**Background Count A:** \_\_\_\_\_, **Background Count B:** \_\_\_\_\_,  
**Background Coincidence Count:** \_\_\_\_\_

5. Switch the laser on. Work at the laser output power  $\sim 25$  mW.
6. The counts should now have increased. If count is low, maximize the counts by adjusting the ‘X-Y’ and z translators on the mounts of collecting system. Once you have maximized the counts you should have at least a hundred coincidences.
7. Switch the APDs off.
8. Put Polarizers A and B at  $0^\circ$ .

## **7. FIRST MEASUREMENTS OF SINGLES AND COINCIDENCES**

**Next step (you will use an Excel template)**

1. While Polarizers A and B are at  $0^\circ$ , record the singles counts on A and B and coincidences.
2. Next put the polarizers A at  $10^\circ, 20^\circ, \dots, 360^\circ$  and each time record Count A and Count B and coincidence count and note them in an Excel table. Make 3 measurements for coincidence count at each angle of a polarizer A and select average.
3. Remove the Polarizer B and put Polarizer A at  $10^\circ, 20^\circ, \dots, 360^\circ$  and each time record the 3 coincidence count measurements, define the average value and note it down in Excel.

4. Plot Count A, Count B and Coincidence count (on a separate plot) versus the polarization angle of Detector A.

The plots for the singles counts are not exactly flat, as you should have predicted. The plots seem to have some sinusoidal characteristic. We define a quantity called **Visibility (V)** to check the flatness of a curve.

$$V = \frac{Max - Min}{Max + Min} \times 100\% \quad , \quad (11)$$

Here 'Max' and 'Min' are the maximum and minimum counts. Find the 'Max' and 'Min' of your data and calculate the visibility.

**Max:** \_\_\_\_\_, **Min:** \_\_\_\_\_, **Visibility:** \_\_\_\_\_

**QUESTION 7.1:**

You know that you have the quantum state  $\Psi = |H_s H_i\rangle + |V_s V_i\rangle$ . How do you expect Count B to change as you rotate Polarizer B from 0° to 360°, but Polarizer A is set for a fixed angle  $\alpha$ ?

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**QUESTION 7.2:**

If Polarizer A was removed how would you expect the coincidence count to change as you rotate Polarizer B from 0° to 360° ?

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**QUESTION 7.3:**

What would you expect the visibility to be for a perfectly flat curve?

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### QUESTION 7.4:

What does it mean to have a 100% visibility?

## 8. LOOKING AT THE ENTANGLEMENT

In this activity we will see how photons in two arms are entangled and how this entanglement affects the properties of one photon when a measurement is performed on the other photon.

### QUESTION BEFORE STARTING:

What would you expect for the Coincidence Count if you put Polarizer A at  $\alpha = 0^\circ, 45^\circ, 90^\circ, 135^\circ$  and in each case rotate Polarizer B from  $\beta=0^\circ$  to  $360^\circ$ ? Show the mathematical basis for your prediction. (Hint: fixing  $\alpha$  at some angle is like making a measurement in that basis).

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### Next steps to check your suggestions:

1. Set the Labview program for a measurement time of 1-5 seconds (or 1000-5000 ms)
2. Fix polarizer angles  $\alpha = 0$  and put  $\beta = 0$ . Switch the lights off. Make sure that the door is locked. Make sure that the laser is off.
3. Ask the TA to switch on the APD.
4. Record the coincidence count by clicking "START" in the Labview program.
5. Next put the Polarizer A at  $\alpha = 10^\circ, 20^\circ, \dots, 360^\circ$ . Record the coincidence count and note the results (you did these measurements in a previous section).
6. Repeat step 5 for  $\beta = 45^\circ, 90^\circ, 135^\circ$  and note down the counts in your Excel table.
7. Plot Coincidence Counts for the different  $\beta$  values versus the angle of a polarizer  $\alpha$ . (There should be four plots, one for each value of  $\beta$ ).

This dependence you observed is actually the evidence of quantum entanglement. You should have seen that the maximum coincidence

count occur at the angle where  $\alpha$  has been fixed. (For  $\alpha=0$ , you get the maximum of coincidence at  $\beta=0$ . For  $\alpha=90$ , you get the maximum of coincidence at  $\beta=90$ , etc.). But for entanglement you also should measure fringe visibility  $V$  (equation (11)). If you have  $V > 0.71$  for all these curves, you have an entanglement.

**Question 8.1:**

What equation of the section 4 matches the experimental results proving the entanglement?

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**Question 8.2:**

Compare the coincidence plot with Polarizer A removed to the plots at different values of  $\alpha$ . What is the difference between coincidence plot with no polarizer and coincidence plot with the polarizer at  $\alpha=0$ ?

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**Question 8.3:**

How does choosing different values of  $\alpha$  change the coincidence plots than  $\beta$  is fixed?

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## **9. TESTING QUANTUM THEORY BY BELL'S INEQUALITY VIOLATION**

In this activity, we will mathematically check if quantum theory is correct by calculating Bell's Inequality in a form of Clauser, Horne, Shimony, and Holt [4, 7]. In the previous activity, we saw evidence of entanglement. In this activity we will try

to perform a mathematical check by violating Bell's Inequality. (Look at the Refs 7, 9-11 for an explanation of Bell's inequality.) The maximum possible value of  $S$  in Bell's Inequality is always less than  $|2|$ , for any classical correlation. Getting a value greater than  $|2|$  for  $S$  would confirm the violation of this inequality.  $S$  is defined as follows [4, 7]:

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b') \quad (12)$$

We will calculate the value of  $S$  for

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) - N(\alpha, \beta_{\perp}) - N(\alpha_{\perp}, \beta)}{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) + N(\alpha, \beta_{\perp}) + N(\alpha_{\perp}, \beta)} \quad (13)$$

$N(\alpha, \beta)$  is the coincidence count when Polarizer A is at  $\alpha$  and Polarizer B is at  $\beta$ .

1. Set the Labview program for a measurement time of 5 seconds (or 5000 ms).
2. Switch the lights off. Make sure that the door is locked. Make sure that the laser is off.
3. Ask the TA to switch on the APDs.
4. Put Polarizer A at  $\alpha = -45^{\circ}$  and Polarizer B at  $\beta = -22.5^{\circ}$ .
5. Record the coincidence count by hitting "START" in the Labview program and note it down in your Excel template for Bell's inequality. Make 3 measurements for coincidence count at each angle setting and calculate average coincidence in each setting.
6. Repeat this until you have collected data for all of the Table 1.
7. Measure singles count as well at each angle setting. Calculate accidental coincidences and remember this equation # 14:



$$N_{\text{accidental}} = (\text{Singles A}) \times (\text{Singles B}) \times 26 \text{ ns (accuracy of time measurements)} / (\text{accumulation time of 1 or 5 s}) . \quad (14)$$

8. Subtract accidental coincidences from average coincidences at each angle set.

9. For  $a=-45^\circ$ ,  $a'=0^\circ$ ,  $a_\perp=45^\circ$ ,  $a'_\perp=90^\circ$ ,  $b=-22.5^\circ$ ,  $b'=22.5^\circ$ ,  $b_\perp=67.5^\circ$ ,  $b'_\perp=112.5^\circ$ , calculate  $S$  in Equation (12) and note it down. An Excel template will calculate  $S$  automatically.

**Table 1:**  $\alpha$  = polarization angle of Polarizer A

$\beta$  = polarization angle of Polarizer B

Net coincidence count = Average coincidence count – Accidental coincidences. These accidental coincidences result from the probability that the two uncorrelated photons from two different down-conversion events will arrive within the coincidence interval. This background is small and acts to decrease  $|S|$ . A finding  $|S|>2$  thus cannot be a result of the accidental coincidences.

Accidental coincidences -			
$\alpha$ (deg)	$\beta$ (deg)	Coincidence Count	
		Average	Net
-45	-22.5		
-45	22.5		
-45	67.5		
-45	112.5		
0	-22.5		
0	22.5		
0	67.5		
0	112.5		
45	-22.5		
45	22.5		
45	67.5		
45	112.5		
90	-22.5		
90	22.5		
90	67.5		
90	112.5		

$S =$  \_\_\_\_\_

Violation of Bell's Inequality can be seen at many angle settings. The maximum is observed at the angles chosen above.

**Question 9.1:**

What is the value of experimental error in your measurements of  $S$ ?

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### Question 9.2:

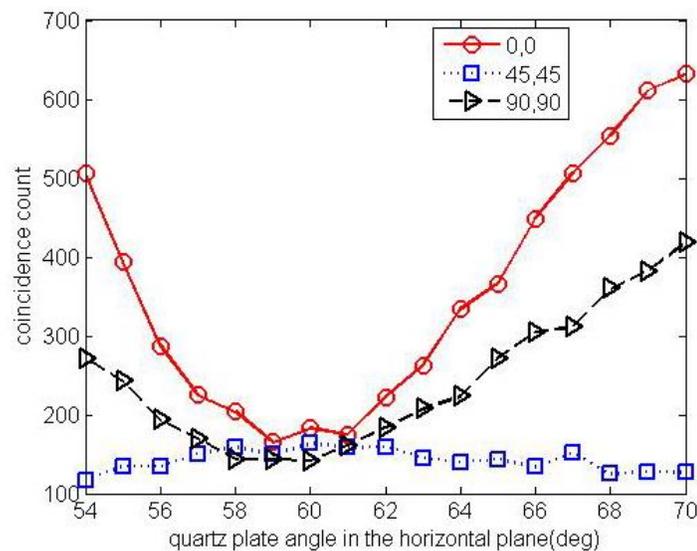
The predicted value of  $|S|$  from quantum theory at the angles you used in this activity is 2.82. Discuss why you did not get this value of  $|S|$  (i.e., what are the sources of experimental errors).

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## 10. ROTATION OF A QUARTZ PLATE AROUND VERTICAL AND HORIZONTAL AXES

To compensate the phase difference between downconverted photons with vertical and horizontal polarizations (one passes through two BBO crystals, another – through only one crystal), the quartz plate should be rotated both in a horizontal plane and a vertical plane. Three coincidence count curves should be plotted, corresponding to  $(\alpha, \beta)$  value of  $(0^\circ, 0^\circ)$ ,  $(45^\circ, 45^\circ)$  and  $(90^\circ, 90^\circ)$  respectively at different angles of rotation of a quartz plate.



**Figure 8 . Coincidence count changes as the quartz plate is rotated around the vertical axis. The angle of rotation around the horizontal axis is fixed (Z. Shi, H. Shin, S. White, N. Savidis).**

The coincidence count at the three polarizer settings changes vastly as the quartz plate rotates either horizontally and vertically. A small change of the quartz plate (e.g., 1 deg.) can change the coincidence counts by more than 20% (see Figure 8). The optimal position of a quartz plate is when the coincidence counts for all settings is equal (intersection of the curves).