Experimental Violation of Bell's Inequalities through Polarization Entanglement

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OPT 253 - University of Rochester, 2018

The phenomenon of quantum entanglement is a phenomenon whose understanding eluded the world's greatest scientists for decades past its discovery. Einstein himself famously rejected the exact mechanism by which it seems to operate, poking fun at this seemingly nonsensical explanation by referring to it as "spooky actions at a distance". However, following the formulation of Bell's Inequalities, proving this mechanism became a matter of experiment. This lab seeks to provide such experimental evidence for entanglement through observations of polarization-entangled photons. Three separate experiments are performed to this end. Each of the three experiments utilize a BBO crystal to produce two polarization-entangled photons from one input photon. Each of these two photons are then measured at various angles by a polarizer and a detector capable of registering the incidence of single photons. The first experiment seeks to collect all necessary data to violate Bell's Inequalities and prove the presence of polarization-entanglement. This was achieved in two separate trials with an *S* value of 2.472 and 2.564, respectively. The second experiment then measures the relationship between coincidence counts of the two entangled photons, and the relative angle between the polarizers measuring them. This was achieved for four separate trials, each indicating a cosine-squared dependence each with visibility of V>0.707, as predicted. The third experiment improves upon the phase-matching conditions of the previous two experiments by use of a quartz plate. This was achieved and the resultant *S* value saw an improvement of 0.127 from its previous measurement. The results of all three experiments are analyzed and found to agree with the nonlocal hypothesis of quantum entanglement.

I. Theory

<u>Entanglement</u>

The principle phenomenon studied in these experiments is known as *Quantum Entanglement*, or, as Einstein so famously put it: "spooky actions at a distance". This phenomenon was first introduced in the 1935 thought experiment by Einstein, Podolsky and Rosen[1] (EPR), in which they claimed to demonstrate a paradox in the *Copenhagen Interpretation* of quantum mechanics due to the interactions of *entangled* particles that quantum mechanical formulations had hitherto overlooked. *Entanglement* between particles occurs anytime that the state of two or more particles cannot be described as the product of their independent states, but must be described by an *ensemble state*, which represents both particles in one wavefunction. A natural consequence of entanglement ~~between~~ particles means that the measurement of the state of one particle will instantaneously affect the state of its partners, no matter the distance between them.

[1] Einstein, A., Podolsky, B., & Rosen, N. (1935).
Can quantum-mechanical description of physical reality be considered complete?. *Physical review*, *47*(10), 777.

This conclusion leads to the vexing implication of *nonlocality*, whereby an action made in one location can affect the outcome of an action that is nonlocal, an action that is arbitrarily far away. This conclusion does not, however, imply the propagation of information faster than the speed of light and consequently does not necessitate a violation of *special relativity*, as the measurement of each entangled particle will appear random to the party measuring it, although upon further inspection of all outcomes, each random measurement will be correlated with every other random measurement. The resolution to this alleged paradox, according to EPR, was the formulation of *hidden variables* that dictate the outcome of each measurement of the entangled particles. However, the interpretation invoking hidden variables yields a statistical outcome that differs from the outcome ~~that~~ yielded by the nonlocal interpretation, as can be seen in the formulation of Bell's inequalities.

Bell's Inequalities

John Stewart Bell was a 20th century theoretical physicist and mathematician that, upon hearing of the EPR paradox, wrote a response in 1964 entitled "On the Einstein-Podolsky-Rosen Paradox" in which he formulated a series of testable statistical quantities that would determine, one way or the other, if the hidden variable or nonlocal interpretations reflected observable reality.[2]

The exact calculation of these inequalities will be covered briefly in the following section, however the intuition underpinning this formulation comes from the differing statistics for correlated and uncorrelated events. Similar to the calculation of correlated and uncorrelated error: for maximally correlated events, the total probability is simply the sum of the probabilities of the two events. For maximally *un*correlated events, the total probability is the sum of the two independent events added *in quadrature*. If, for example, the probability for a single event is √2, then the total probability, if these events are maximally uncorrelated, is 2, whereas the total probability, if these events are maximally correlated, is 2√2. This is in fact the exact case we are considering in this experiment, where S is the variable analogous to the total probability of two events. Bell's inequalities simply state that if $S>2$ for two events, the two events must be correlated to some extent, with the understanding that $S=2\sqrt{2}$ represents the case of maximum correlation between events.

As applied to the EPR Paradox, measuring the state of two entangled particles with a value of $S>2$ would indicate that the states *are* correlated with each other, implying that the measurement of one *does* seem to impact the measurement of it partner. A value of $S>2$ would therefore support the nonlocal hypothesis, as opposed to the hidden variables hypothesis, as proposed by EPR.

Polarization Entanglement

In order to test Bell's inequalities experimentally, both the particle and the state that is to be entangled between particles must first be chosen. Here, we have chosen to entangle and measure *polarization* states of two entangled *photons* created by a process known as *Optical Parametric Down Conversion* (OPDC). OPDC is a nonlinear process by which a single photon of frequency ω_0

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[2] Bell, J. S. (1964). On the Einstein Podolsky Rosen Paradox. Physics, 1 (3), 195-200.

traveling through a nonlinear birefringent crystal is spontaneously down-converted (with an efficiency of ~10^{-10}) to a pair of two photons of smaller frequency, ω_1 and ω_2, such that $\omega_1 + \omega_2 = \omega_0$. The process is called "parametric" because the initial and final energy states are identical:

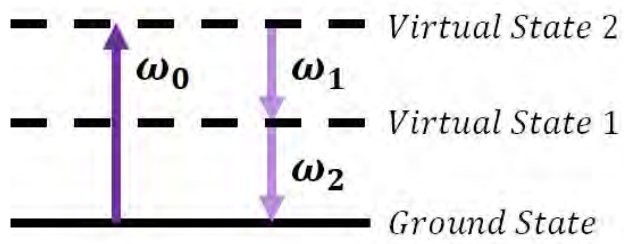

Figure 1: A schematic depicting the energy states during OPDC in a nonlinear crystal. An upward arrow indicates a photon that is absorbed and a downward arrow indicates a photon that is emitted.

In our specific case: $\omega_1 = \omega_2 = \omega_0/2$, and the net result is two beams each with frequency $\omega_0/2$ and wavelength $2\lambda_0$. By momentum conservation, the two down-converted photons must exit the crystal at opposite angles, traditionally denoted as *signal* and *idler* beams. By orienting the *optic axis* of the crystal in a particular manner, the two down-converted photons can be made to each have polarizations perpendicular to the input polarization (known as Type I phase-matching). If the orientation of the crystal is chosen such that horizontal polarization, $|H\rangle$, produces a signal and idler beam of vertical polarization, $|V\rangle$, denoted:

$$|H\rangle \rightarrow |V_s V_i\rangle$$

the same process can then be replicated for the case:

$$|V\rangle \rightarrow |H_s H_i\rangle$$

by rotating the crystal 90°. By using two (optically contacted) crystals, with one rotated at 90° with respect to the other and inputting 45° polarization, the case:

$$\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) \rightarrow \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i) \quad \textbf{(Eq. 1.1)}$$

can be achieved. In this case, the final polarization state is non-separable, meaning it represents a case of *polarization entanglement*. By measuring the polarization of this state for a number of angles and evaluating *S*, Bell's inequalities can be violated, indicating the presence of polarization entanglement and supporting the nonlocal hypothesis.

In this formulation, it can be shown that S is calculated by the following:[3]

$$S = E(a-45°, b-22.5°) - E(a-45°, b+22.5°) + E(a, b-22.5°) + E(a, b+22.5°) \quad \textbf{(Eq. 1.2)}$$

where a and b represent the angles of the polarization state to be measured of the signal and idler beams, respectively and

[3] Professor Svetlana G. Lukishova (2018). OPT 253 Lab 1 Manual: *Entanglement and Bell's Inequalities.*

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha{+}90°, \beta{+}90°) - N(\alpha, \beta{+}90°) - N(\alpha{+}90°, \beta)}{N(\alpha, \beta) + N(\alpha{+}90°, \beta{+}90°) + N(\alpha, \beta{+}90°) + N(\alpha{+}90°, \beta)} \quad \textbf{(Eq. 1.3)}$$

with $N(\alpha, \beta)$ representing the number of *coincidence counts* (measurements for which two photons hit the two detectors used to measure them simultaneously) measured for each set of polarization state angles.

Cosine-Squared Dependence and Visibility

Due to *Malus' law*, measuring the polarization of light using an ideal polarizer will yield an intensity related to the cosine of the angle between the polarizer and incident state, θ, squared:

$$I = I_0 cos^2\theta \quad \textbf{(Eq. 1.4)}$$

where I is the measured intensity and I_0 is the input intensity. If we extend this relation to not only individual measurements of polarization but to the coincidence counts of the two down-converted photons, we will see that for the entangled states:

$$N(\alpha, \beta) = \frac{A}{2}\ cos^2(\alpha - \beta) \quad \textbf{(Eq. 1.5)}$$

where A represents the total number of entangled pairs produced via SPDC. As with any cosine-squared dependence, the visibility can be calculated as

$$V = \frac{N_{max} - N_{min}}{N_{max} + N_{min}}\ . \quad \textbf{(Eq. 1.6)}$$

The value of S is therefore limited by the measured visibility: $S = V S_{max}$ where $S_{max} = 2\sqrt{2}$ and so a visibility of $V > \frac{1}{\sqrt{2}} = 0.707$ must be measured in order to yield $S > 2$, and successfully violate Bell's inequalities and demonstrate the presence of polarization entanglement.

II. Experimental Procedure

The procedures in this lab were divided into three individual experiments, each dedicated to investigating a different aspect of the theory behind polarization entanglement but all sharing the same experimental setup:

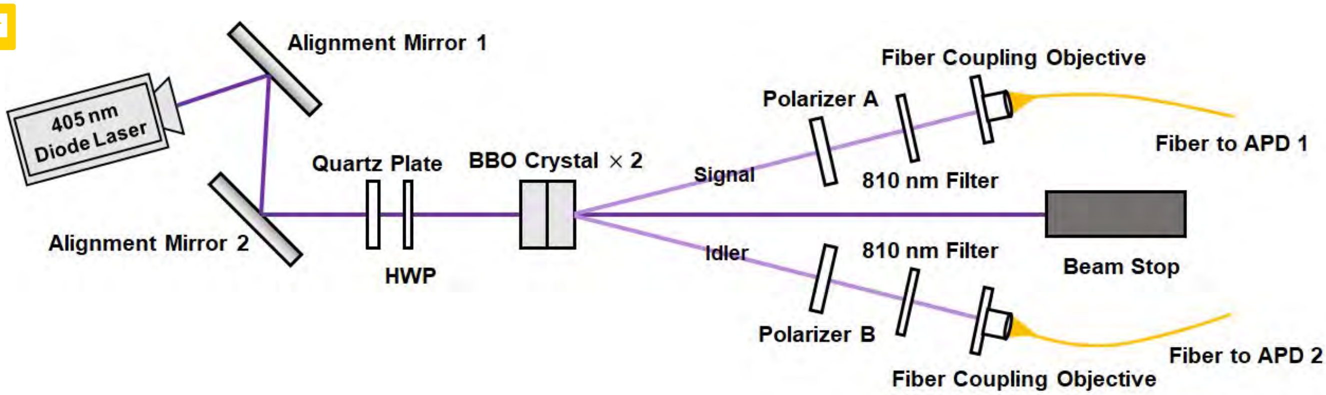

Figure 2.1: A schematic displaying the basic experimental setup used for each experiment. A 405 nm diode laser operated at 35mW is aligned by two alignment mirrors, polarized at 45 ° by the half-wave plate (HWP), corrected for phase by the quartz plate and sent through the pair of optically contacted Type I BBO crystals to undergo SPDC. The majority of the beam is not converted and is absorbed by the Beam Stop, while the down-converted signal and idler beams continue on to be polarized by their respective polarizers (A and B), filtered at 810 nm ($2\lambda_0$) and fed into one of two avalanche photodiodes (APDs) by use of an optical fiber and fiber coupling objective.

The output of the APDs is sent to a coincidence counter that records the arrival of simultaneous photons at APD 1 and 2. The coincidence counter has an effective response time of 26 nanoseconds, and each measurement of coincidence counts was aggregated over an acquisition time of 1 second.

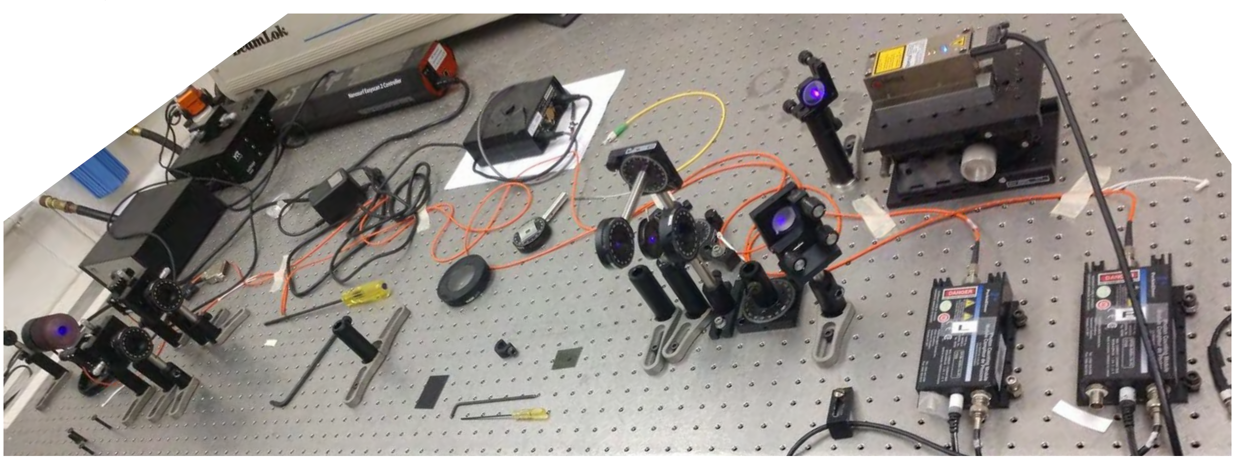

Figure 2.2: A photo of the actual lab setup used for Lab 1. Here, the 405 nm diode laser is on, and can be seen in the upper right. The beam is seen reflecting off the two alignment mirrors and entering the pair of BBO

crystals in the center of the image. On the far left, the beam stop is visible as well as the two series of polarizers, filters, and fiber coupling objectives that feed back into the two APDs in the lower right.

Experiment 1: Violating Bell's Inequalities

This portion of the lab was dedicated to calculating a value of S (according to Eq. 1.2) greater than the classical limit of S=2. To do this, measurements of coincidence counts between APD 1 and 2 were taken for a discrete set of angles, as dictated by Eq. 1.2 and 1.3.

Part 1) For the initial polarizer angles of a=b=0°, violation of Bell's inequalities require measurements at the Polarizer A angles of α = -45°, 0°, 45°, and 90° where, in each case, Polarizer B is then rotated through β = -22.5°, 22.5°, 67.5°, and 112.5°.

Part 2) In order to demonstrate that Bell's Inequalities can only be violated using a discrete set of angles, we then took two sets of measurements using angles that did *not* correspond to those in Eq. 1.2 and 1.3. The first set consisted of Polarizer A ranging from α=0 to 160° in intervals of 10°, and Polarizer B at β=0°. The second set consisted of Polarizer A and B both assigned semi-random angles.

Part 3) To demonstrate that Bell's inequalities can be violated for any basis so long as the measurements adhere to Eq. 1.2 and 1.3, we then took another set of measurements with initial angles a=b=45°, resulting in Polarizer A at α=0°, 45°, 90°, and 135°, while Polarizer B was rotated through β = 22.5°, 67.5°, 112.5°, and 157.5°.

Note: The quartz plate shown in Figure 2.2 was not used in this portion of the experiment.

Experiment 2: Measuring Cosine Squared Dependence

The purpose of this portion of the lab was to collect enough data to demonstrate an ideal cosine-squared dependence of coincidence counts on the relative angle of the two polarizers, as stated in Eq. 1.5. In order to show that this cosine-squared dependence is present for all bases and invariant of rotation, this experiment was replicated at four different values of Polarizer A angle, α=0°, 45°, 90°, 135°, where, in each case, Polarizer B was swept through the angles β = 0 to 360° by increments of 10°. From this trend, visibility can then be calculated in order to show that it exceeds the classical limit of V=0.707.

Note: The quartz plate shown in Figure 2.2 was not used in this portion of the experiment.

Experiment 3: Correcting for Phase Offset

In Eq. 1.5, the relative phase offset between horizontal and vertical polarization is assumed to be 0. However, in each experiment, the distance between the optically contacted BBO crystals introduces a small phase offset between vertical and horizontal polarizations that may cause a decrease in the visibility of the expected cosine-squared trend. Ideally, the quartz plate shown in Fig. 2.2 would be used to correct for this phase, however, this plate was not used in Experiment 1 and 2.

Therefore, in this part of the lab, the ideal horizontal and vertical rotations of the quartz plate were determined in order to maximize the visibility of our expected trend. This was achieved by determining the horizontal and vertical angles (of the quartz plate) at which the coincidence counts for APD 1 and 2 were most closely equal. In each case, Polarizer A and B were parallel and rotated through $\alpha = \beta = 0°, 45°, 90°$. For the horizontal rotation, the quartz plate was rotated through -14° to 13° at intervals of 1°. For the vertical rotation, the plate was set at the best horizontal angle and then rotated vertically through -70° to 70° at intervals of 10°.

After determining the ideal values for both horizontal and vertical rotation, the quartz plate was set to these values and *S* was recalculated using the same procedure as in Experiment 1, Part 1.

III. Results and Data Analysis

Experiment I: Violating Bell's Inequalities

Throughout this lab, the possibility of detecting *accidental coincidences* (coincidences that do not reflect the presence of entangled photons) introduces an amount of error in every measurement of coincidences recorded. Because this error scales with the total number of coincidences recorded on each detector, we can estimate the number of accidental coincidences that were likely recorded for each measurement, and subtract away this error to yield cleaner results. Equation 3.1 roughly estimates the number of accidental coincidences, N_{acc}, that are recorded for each measurement:

$$N_{acc} = N_A N_B \frac{\tau}{\Delta T} \qquad \textbf{(Eq. 3.1)}$$

where N_A and N_B are the *singles* count (the number of counts on each APD) for APD 1 (corresponding to Polarizer A) and 2 (corresponding to Polarizer B), τ is the response time of the coincidence counter (26 ns) and ΔT is the acquisition time of each measurement (1 second).

Part 1) Coincidence counts for **a=b=0°**

Polarizer Angle of Detector A (°)	Polarizer Angle of Detector B (°)	Net Coincidences	$E(\alpha,\beta)$
$\alpha = -45$	$\beta = -22.5$ $\beta = 22.5$ $\beta = 67.5$ $\beta = 112.5$	151.56 40.00 18.61 136.10	0.7995
$\alpha = 0$	$\beta = -22.5$ $\beta = 22.5$ $\beta = 67.5$ $\beta = 112.5$	127.03 92.86 20.43 33.91	0.6396
$\alpha = 45$	$\beta = -22.5$	8.28	0.5134

	β = 22.5 β = 67.5 β = 112.5	68.56 89.82 24.69	
α = 90	β = -22.5 β = 22.5 β = 67.5 β = 112.5	45.08 14.19 76.69 125.95	-0.5196
S = 2.472			

Table 3.1: The angles of Polarizer A and B for each measurement taken, the corresponding coincidence counts, and the value of $E(\alpha,\beta)$ (as seen in Eq. 1.3) calculated using this data. The final value of S (as seen in Eq. 1.2) is displayed at the bottom of the table.

As seen in the table above, this set of angles produced a value of S>2, therefore violating the classical limit of Bell's Inequalities and demonstrating polarization entanglement.

Part 2 & 3) Coincidence counts for $\boldsymbol{\alpha}$ **= 0° → 160°,** $\boldsymbol{\beta}$ **= 0°**
Coincidence counts for **random angles**
Coincidence counts for **a=b=45°**

Method	S	S>2 ?
α = 0° → 160° β = 0°	0.294	No
Random	0.463	No
α = 0°, 45°, 90°, 130° β = 22.5, 67.5, 112.5, 157.5	2.564	Yes

Table 3.2: The method by which data was taken, the calculated value of S (as seen in Eq. 1.2), and whether or not the method produced results to violate Bell's Inequalities.

As seen in the table above, the first two methods failed to prove a violation of Bell's Inequalities. The third method, however, did. Whereas the first two methods did not measure the set of angles as dictated by Eq. 1.2 and 1.3, they were treated as if they had, and so the resultant value of S is simply a meaningless statistical quantity that does not adhere to the classical limit of S=2, let alone represent a case where S>2. The angles measured in the third method however, *do* adhere to Eq. 1.2 and 1.3, but are simply a rotated basis where the initial angles (a and b) were chosen arbitrarily at 45°, indicating that Bell's Inequalities can, in fact, be violated for an arbitrary rotated basis.

<u>Experiment II: Measuring Cosine Squared Dependence</u>

Cosine-squared trend for **α=0°, 45°, 90°, and 135°**

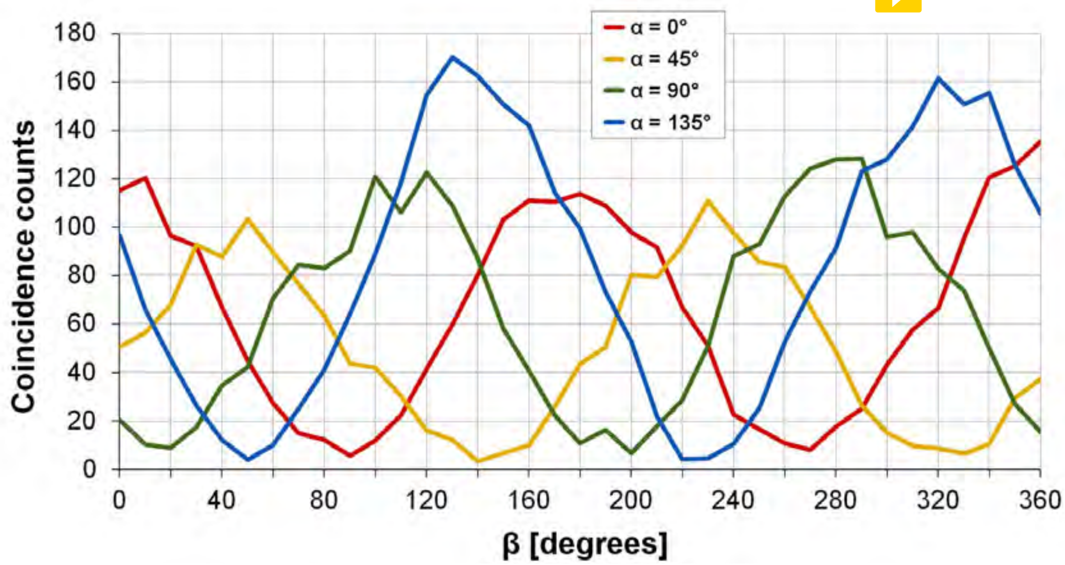

Figure 3.1: Four plots of coincidence counts as a function of Polarizer B angle, β, each corresponding to a different value of Polarizer A angle, α. Note that each plot displays a cosine-squared-like dependence, each shifted relative to the first case (α=0°) by approximately α degrees.

As can be seen in the above graph, each plot roughly follows the cosine-squared trend as predicted by Eq. 1.5, modulating as a function of both α and β. However, the amplitude should ideally be uniform for each value of α, but it is clearly larger for the case that α=135°. As noted above, this is likely due to the phase offset introduced by the physical distance between the two BBO crystals, as this was not corrected for by use of a quartz plate.

We can now calculate the visibility of each trend, using Eq. 1.6:

Angle of Polarizer A (Degrees)	**Visibility**
$\alpha = 0$	0.85
$\alpha = 45$	0.84
$\alpha = 90$	0.94
$\alpha = 135$	0.95

Table 3.3: The visibility, V, calculated for each series of data shown in Fig. 3.1 using Eq. 1.6.

Upon inspection, we can see that although visibility varies with α (when in the ideal case, it would be independent of α), each value still exceeds the condition established previously that $V>0.707$, again surpassing the classical limit and fulfilling the necessary criteria to violate Bell's Inequalities.

Experiment III: Correcting for Phase Offset

Coincidence counts for **horizontal rotation** of the quartz plate from -14 through 13°:

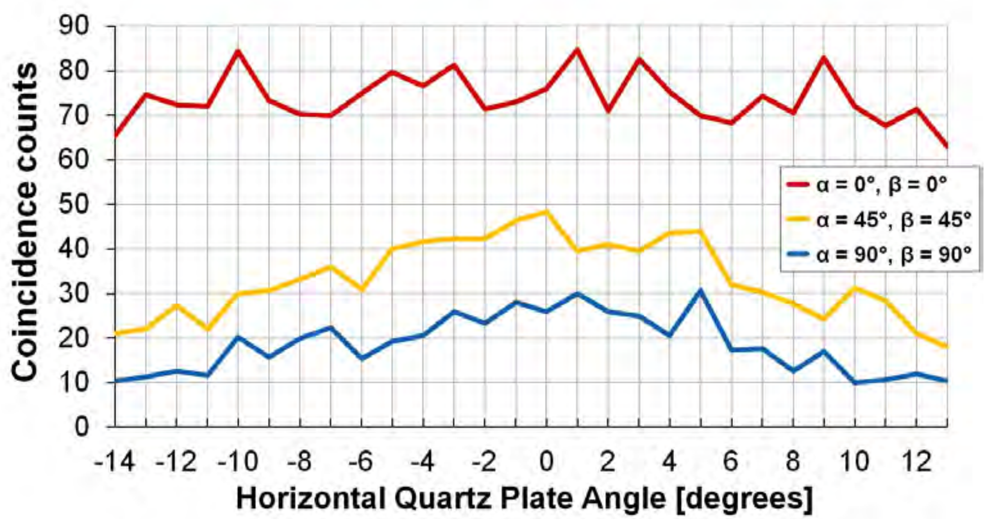

Figure 3.2: A plot of the coincidence counts as a function of the horizontal quartz plate angle for three cases where α=β=0°, 45°, and 90°.

Ideally, all three coincidence counts, for some value of the rotation angle, would be equal, however, as the system had yet to be corrected for vertical rotation when this data was taken, it would seem that horizontal rotation alone was evidently insufficient to significantly improve the phase offset. It was ultimately decided that 9° of horizontal rotation represented the best agreement between the α=β=45° and 90° cases, and so this value was chosen before correcting for vertical rotation.

Coincidence counts for **vertical rotation** of the quartz plate from -70 through 70°:

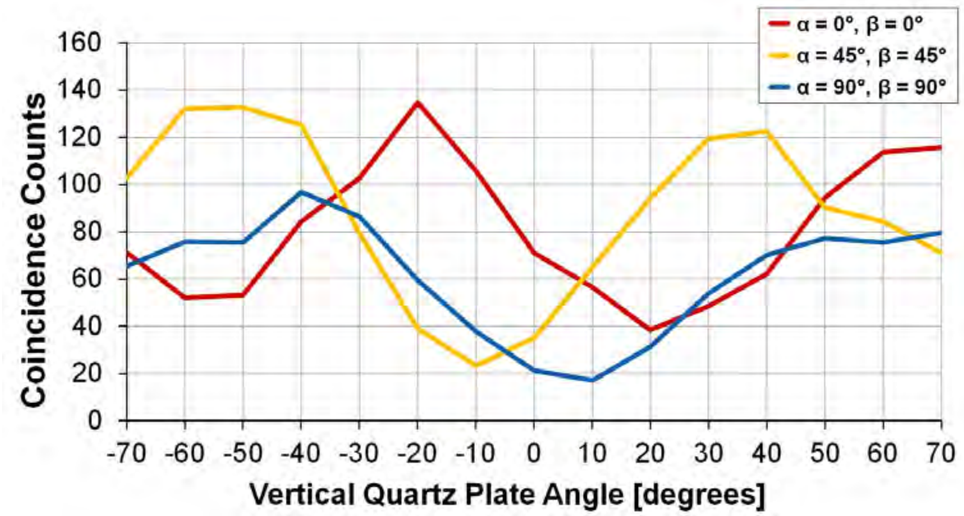

Figure 3.3: A plot of the coincidence counts as a function of the vertical quartz plate angle for three cases where $\alpha=\beta=0°$, 45°, and 90°.

In this case, the three coincidence counts intersect at multiple points, however it was agreed that the best agreement between all three occurred at approximately -34°. Thus, the quartz plate was rotated to a horizontal angle of 9° and a vertical angle of -34° before again measuring coincidence counts as in Experiment 1, Part 1.

Coincidence counts for **a=b=0°**

Polarizer Angle of Detector A (Degrees)	Polarizer Angle of Detector B (Degrees)	Net Coincidences	$E(\alpha,\beta)$
α = -45	β = -22.5 β = 22.5 β = 67.5 β = 112.5	93.02 23.10 11.50 82.34	0.84
α = 0	β = -22.5 β = 22.5 β = 67.5 β = 112.5	74.39 57.27 15.33 18.51	0.64
α = 45	β = -22.5 β = 22.5 β = 67.5 β = 112.5	5.19 70.87 99.90 24.77	0.59
α = 90	β = -22.5 β = 22.5	28.70 16.51	-0.52

	β = 67.5 β = 112.5	98.01 103.07	
S = 2.599			

Table 3.4: The angles of Polarizer A and B for each measurement taken, the corresponding coincidence counts, and the value of $E(\alpha,\beta)$ calculated using this data. The final value of S is displayed at the bottom of the table.

After adjusting the horizontal and vertical orientation of the quartz plate, our final value of *S* for the case of a=b=0° increased by 0.127 from the previous measurement of 2.472, representing the largest value of S calculated in all three experiments.

IV. Conclusion

Each of the three experiments conducted in this lab provide a unique insight on the concept of polarization entanglement, that, when considered together, have broad implications for the field of quantum mechanics at large.

Experiment 1 successfully demonstrated a violation of Bell's inequalities on two accounts: using the basis of a=b=0°, yielding an *S* value of 2.472, and using the rotated basis of a=b=45°, yielding an even larger *S* value of 2.564. Both of these values are well above the classical limit of *S*=2, providing experimental evidence for the presence of quantum entanglement and, more broadly, supporting the nonlocal hypothesis that was originally rejected by EPR.

This result was then further supported by the findings of Experiment 2, which provided visible evidence that polarization states are correlated between entangled photons, with a visibility of this correlation far above the classical limit of *V=0.707* for four separate trials, each in a new rotated basis. The fact that this trend was independent of the rotation basis it was measured in lends credence to the idea that quantum entanglement is a fundamental phenomenon of quantum mechanics and *not* simply a statistical consequence of how states are measured in lab.

Experiment 3 served to demonstrate that the small variations between results measured as measured in Experiment 2 are likely only a consequence of imperfect phase-matching conditions in lab. The calculated value of *S* improved significantly (by as much as 0.127) after correcting for phase more precisely. This correction is a clear indication that, in ideal conditions, an exact value of $S=2\sqrt{2}$ should be measured for all rotation bases, in exact agreement with Bell's theorem.

Taken together, all three experiments use the case for polarization entanglement between two photons to support the more general idea that quantum entanglement between particles is a *real* phenomenon. The notion of *nonlocality*--a notion that Einstein himself disregarded as impossible--is, in fact, observable. It is yet another aspect of quantum mechanics that may seem inexplicable in terms of a more intuitive classical reality, but it is a verifiable aspect of the exceedingly strange quantum world. As Feynman so cleverly put it: *"The difficulty really is psychological and exists in the perpetual* *torment that results from your saying to yourself, "But how can it be like that?" which is a reflection of uncontrolled but utterly vain desire to see it in terms of something familiar...I think I can safely say that nobody understands quantum mechanics...I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing. Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get 'down the drain', into a blind alley from which nobody has escaped. Nobody knows how it can be like that."*[4]

Contribution of Each Student

Andrew Howard - *Theory, Experimental Procedure*
Kyle Guzek - *Results and Data Analysis*
Both - *Abstract, Conclusion*

References

[1] Einstein, A., Podolsky, B., & Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete?. *Physical review*, *47*(10), 777.

[2] Bell, J. S. (1964). On the Einstein Podolsky Rosen Paradox. Physics, 1 (3), 195-200.

[3] Professor Svetlana G. Lukishova (2018). OPT 253 Lab 1 Manual: *Entanglement and Bell's Inequalities.*

[4] Richard P. Feynman, (1964). *The Messenger Lectures*, MIT.

Total: 96/100

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1)Abstract:	**9/10**
2)Conceptual Understanding	**14/15**
3)Experimental Procedure Description	**14/15**
4)Measurement Results	**20/20**
5)Error and Uncertainty	**5/5**
6)Figures and Data Representation	**20/20**
7)Language Style and Accuracy	**14/15**
8)Extra Points	**0**

Comments:

Overall, this report was very well done, though a few things could have been improved.
Your abstract was solid, though in places omitted details that are important for a succinct but *complete* description of the work that was done.
It is evident you have a strong conceptual understanding, but the presentation of that understanding sometimes summarized the chain of logic too briefly, or didn't explicitly define all quantities being used.
The experimental details were described well, but like the theory, there were places where details that the reader needs to know were omitted or not presented explicitly.
The vocabulary used was colorful and appropriate, and made for a great presentation, but there were places in the formatting (specifically surrounding figures and diagrams) that could have been clear, and justified body text rather than left aligned would have been preferred as it is a general convention and looks cleaner.

[4] Richard P. Feynman, (1964). *The Messenger Lectures*, MIT.