Quantum Entanglement and Bell’s Inequality

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Abstract – Christopher Marsh: Entanglement is a phenomenon where two particles are linked by some sort of characteristic. A particle such as an electron can be entangled by its spin. Photons can be entangled through its polarization. The aim of this lab is to generate and detect photon entanglement. This was accomplished by subjecting an incident beam to spontaneous parametric down conversion, a process where one photon produces two daughter polarization entangled photons. Entangled photons were sent to polarizers which were placed in front of two avalanche photodiodes; by changing the angles of these polarizers we observed how the orientation of the polarizers was linked to the number of photons coincident on the photodiodes. We dabbled into how aligning and misaligning a phase correcting quartz plate affected data. We also set the polarizers to specific angles to have the maximum S value for the Clauser-Horne-Shimony-Holt inequality. This inequality states that S is no greater than 2 for a system obeying classical physics. In our experiment a S value of 2.5 ± 0.1 was calculated and therefore in violation of classical mechanics.

1. Introduction – Graham Jensen

We report on an effort to verify quantum nonlocality through a violation of Bell’s inequality using polarization-entangled photons.

When light is directed through a type 1 Beta Barium Borate (BBO) crystal, a small fraction of the incident photons (on the order of $10^{-10}$) undergo spontaneous parametric downconversion. In spontaneous parametric downconversion, a single pump photon is split into two new photons called the signal and idler photons. In order to maintain a conservation of energy and momentum, these photons will have longer wavelengths than the pump, i.e., less energy, and have paths deflected at an angle relative to the pump photon propagation direction. Another result of spontaneous parametric downconversion is that the signal and idler photons will be entangled in polarization: if the polarization state of one photon is measured, their entangled wavefunction collapses and the state of the other is known with absolute certainty, no matter the distance between them [1]. Entanglement in polarization can be experimentally measured through a violation of Bell’s inequality.

Using polarization-entangled photons, an experiment to violate Bell’s inequality is performed by measuring the coincidence of entangled photons as a pair of polarizers in their paths are adjusted though various relative polarization differences. If the downconverted photons behave as predicted by a supplemental hidden variable theory, their coincidence counts will not result in a violation of Bell’s inequality. Conversely, if the down-converted photons follow the predictions of quantum mechanics, their coincidence counts will show a contradiction with any hidden variable theory by violating Bell’s inequality.

2. Theory – Graham Jensen

The notion of quantum entanglement was first proposed by Einstein, Podolsky, and Rosen in 1935. The group proposed a paradox (known as the EPR paradox) which was designed to show a flaw in
the current theory of quantum mechanics. David Bohm proposed a simplified version of this situation where a pi meson decays down to an electron and positron. Quantum theory predicts that if the spin of one of the particles is measured the other must be the opposite, no matter the distance between them. Einstein believed this “spooky action at a distance” is in violation of the theory of relativity, citing the impossibility of faster than light information travel. His conclusion was that quantum mechanics is an incomplete theory and there lies a hidden variable that was yet to be discovered [2].

In the years following, several hidden variable theories were proposed to supplement quantum mechanics. However, these theories were dismissed in 1964 when John Bell mathematically proved that any hidden variable theory is incompatible with quantum mechanics [2]. In Bell’s original paper, particles entangled in spin are considered, but, following the generalized formulation by Clauser, Horne, Shimony, and Holt, the same principle can be applied to photons entangled in polarization [2-4].

We may consider the situation seen in Fig. 1. A pump photon undergoes spontaneous parametric downconversion and produces a pair of photons (named signal and idler) with paths leading to polarizers with polarizations of \( \alpha \) and \( \beta \) respectively.

![Fig. 1. A sketch of the proposed situation. An unpolarized pump photon interacts with a type 1 BBO crystal and undergoes spontaneous parametric downconversion (SPDC). As a result of this process, two photons of identical polarization are produced: a signal photon and an idler photon. These photons are directed in separate paths towards polarizers.](image)

If we assume that the realist interpretation of quantum mechanics is true and apply a hidden variable theory, then the outcomes of measurements at each polarizer are independent of each other and are determined only by the angle of their own respective polarizers and a hidden variable, \( \lambda \). In order to satisfy any hidden variable theory, \( \lambda \) will be described by the distribution \( \rho(\lambda) \) with the conditions that \( \rho(\lambda) \geq 0 \) and \( \int \rho(\lambda) d\lambda = 1 \). The detection of vertically and horizontally polarized photons with \( \alpha \) and \( \beta \) bases may be expressed as

\[
A(\alpha, \lambda) = \pm 1 \quad B(\beta, \lambda) = \pm 1
\]

\[
A(\alpha, \lambda) = \pm 1 \quad B(\beta, \lambda) = \pm 1
\]
respectively. The probability of measuring each detectable outcome is given by the following set out integrals:

\[
P_{VV}(\alpha, \beta) = \int \frac{1 + A(\alpha, \lambda) - B(\beta, \lambda)}{2} \rho(\lambda) d\lambda \\
P_{HH}(\alpha, \beta) = \int \frac{1 - A(\alpha, \lambda) + B(\beta, \lambda)}{2} \rho(\lambda) d\lambda \\
P_{VH}(\alpha, \beta) = \int \frac{1 + A(\alpha, \lambda) - B(\beta, \lambda)}{2} \rho(\lambda) d\lambda \\
P_{HV}(\alpha, \beta) = \int \frac{1 - A(\alpha, \lambda) + B(\beta, \lambda)}{2} \rho(\lambda) d\lambda.
\]

where the subscript of \( P \) denotes the direction of detected polarization in the \( \alpha \) and \( \beta \) bases. From here, we may define the function \( E(\alpha, \beta) \) as

\[
E(\alpha, \beta) \equiv P_{VV}(\alpha, \beta) + P_{HH}(\alpha, \beta) - P_{VH}(\alpha, \beta) - P_{HV}(\alpha, \beta) = \int A(\alpha, \lambda) B(\beta, \lambda) \rho(\lambda) d\lambda.
\]

The absolute value function’s property of subadditivity may be utilized to propose the following equations:

\[
|E(\alpha, \beta) - E(\alpha, \beta')| \leq |E(\alpha, \beta)| + |-E(\alpha, \beta')| \\
|E(\alpha', \beta) + E(\alpha', \beta')| \leq |E(\alpha', \beta)| + |E(\alpha', \beta')|
\]

where the apostrophe (‘) denotes a different angle. These equations may be rewritten as

\[
|E(\alpha, \beta) - E(\alpha, \beta')| \leq \left| \int A(\alpha, \lambda) B(\beta, \lambda) \rho(\lambda) d\lambda \right| + \left| - \int A(\alpha, \lambda) B(\beta', \lambda) \rho(\lambda) d\lambda \right|
\]

and

\[
|E(\alpha', \beta) + E(\alpha', \beta')| \leq \left| \int A(\alpha', \lambda) B(\beta, \lambda) \rho(\lambda) d\lambda \right| + \left| \int A(\alpha', \lambda) B(\beta', \lambda) \rho(\lambda) d\lambda \right|
\]

A new value, \( S \), may be defined as

\[
S \equiv |E(\alpha, \beta) - E(\alpha, \beta')| + |E(\alpha', \beta) + E(\alpha', \beta')|
\]

Using the knowledge that \( |A(\alpha, \lambda)| = 1 \), the above equation may be expressed as

\[
S \leq \int (|B(\beta, \lambda) - B(\beta', \lambda)| + |B(\beta, \lambda) + B(\beta', \lambda)|) \rho(\lambda) d\lambda.
\]

But we know that since \( B(\beta, \lambda) = \pm 1 \) and \( B(\beta', \lambda) = 0 \), \( |B(\beta, \lambda) - B(\beta', \lambda)| + |B(\beta, \lambda) + B(\beta', \lambda)| = 2 \). This reduces the integral to the form [3,4]:

\[
S \leq 2 \int \rho(\lambda) d\lambda \quad \Rightarrow \quad S \leq 2.
\]

However, if we assume that the orthodox interpretation of quantum mechanics is true, we may express the state of the downconverted photons as
assuming phase matching between horizontal and vertical components and unpolarized pump photons. $|V\rangle$ and $|H\rangle$ represent the horizontal and vertical polarization states of the signal and idler photon. It is important to note that this wavefunction cannot be factored into single particle states, the fundamental feature of entanglement [5].

When measuring the polarizations through polarizers of angles $\alpha$ and $\beta$ we may express these bases as

$$|V_{\alpha}\rangle = \cos(\alpha) |V\rangle - \sin(\alpha) |H\rangle \quad |H_{\alpha}\rangle = \sin(\alpha) |V\rangle + \cos(\alpha) |H\rangle$$

and

$$|V_{\beta}\rangle = \cos(\beta) |V\rangle - \sin(\beta) |H\rangle \quad |H_{\beta}\rangle = \sin(\beta) |V\rangle + \cos(\beta) |H\rangle.$$ 

If we measure the polarization in this rotated basis we find the same result as the original basis: half of the time both the signal and idler photons are vertically polarized and half the time they are horizontally polarized [5].

The probabilities of coincidence detection for one pair of polarization angles are:

$$P_{VV}(\alpha, \beta) = \left|\langle V_{\alpha}\rangle \langle V_{\beta}\rangle |\Psi_{DC}\rangle\right|^2 = \frac{1}{2} \cos^2(\beta - \alpha) \quad P_{HH}(\alpha, \beta) = \left|\langle H_{\alpha}\rangle \langle H_{\beta}\rangle |\Psi_{DC}\rangle\right|^2 = \frac{1}{2} \cos^2(\beta - \alpha)$$

$$P_{VH}(\alpha, \beta) = \left|\langle V_{\alpha}\rangle \langle H_{\beta}\rangle |\Psi_{DC}\rangle\right|^2 = \frac{1}{2} \sin^2(\beta - \alpha) \quad P_{HV}(\alpha, \beta) = \left|\langle H_{\alpha}\rangle \langle V_{\beta}\rangle |\Psi_{DC}\rangle\right|^2 = \frac{1}{2} \sin^2(\beta - \alpha).$$

These probabilities can be applied to the previously defined function $E(\alpha, \beta)$ and after some manipulation it can be shown that $E(\alpha, \beta) = \cos(2[\beta - \alpha])$. This result may be substituted into the equation for $S$. If the magnitude of the difference in polarization angles is $\pi/8$, i.e., $|\beta - \alpha| = \pi/8$, then the value of $S$ will be at its maximum value, $2\sqrt{2}$. The contradiction in maximum allowed $S$ values between the quantum mechanical and hidden variable theory models confirms their incompatibility [5].

An experimental measurement of the entangled nature of downconverted photons verifies the validity of the nonlocal model of reality. A realizable measurement of coincidence detection probability is achieved by dividing measured coincidences by the total possible number of possible coincidences. This is mathematically expressed as $P_{VV}(\alpha, \beta) = N(\alpha, \beta)/N_{tot}$ [5]. In order to experimentally confirm a violation of Bell’s inequality, a cosine squared dependence of coincidence count versus the magnitude of the difference in polarization angles $\alpha$ and $\beta$ with a visibility of at least 0.71 as well as an $S$ value greater than two must be achieved [4].

3. Experimental Setup and Procedure

3.1 Setup
This figure represents a top-down view of the setup used in Lab 1. The main component missing is the APDs connected to the computer cart [1].

The system used in this lab consists of the components featured in Fig. 2. There is a monochromatic 432 nm wavelength laser. It produces 20 milliwatts of power. The beam is aligned by two mirrors to go into a half-wave plate. The use of the half-wave plate is to ensure the incident beam is polarized to 45 degrees. The beam at this point may go through an aperture (not labeled in Fig. 2) to further ensure it is aligned correctly. Incident photons then reach the quartz plate, which functions as a phase shift corrector for Beta Barium Borate (BBO) crystals. Each crystal is cut in a way that will cause a photon to undergo spontaneous parametric down conversion (SPDC), a process where an incident photon will birth two daughter photons, called signal and idler photons, which are entangled.

As we see in Fig. 3 depending on their orientation, the crystals also produce daughter photons with polarization opposite of the incident beam. In setup used in Lab 1 the two crystals are the same, however before they are glued together one is rotated 90 degrees. In this way the incident photons that are polarized at 45 degrees can go through SPDC and polarized by either crystal.
The signal and idler photons can exit the BBO crystals at many angles, following the laws of conservation of energy and momentum. In this lab we select the angle in which the idler and signal photons exit the same angle, this also means they have the twice the wavelength of the incident beam. Because SPDC can occur at any angle around the optical axis, you can view the exiting daughter photons as a cone of photons as shown in Fig. 4. Fig. 4 shows how there is a phase difference between the cones because the crystals have a thickness. The aforementioned quartz plate is what corrects this phase difference and makes the two cones overlap to one cone of non-polarized light.

SPDC has a low efficiency of $10^{-10}$, a large amount of the incident beam simply goes through the BBO crystals, and runs into the beam stop. The signal and idler photons go through well aligned polarizers, which are used to try to violate Bell’s inequality. The beams continue through to a filter that rejects light in the visible spectrum and then to a focusing lens which will send the photons to an avalanche photodiode (APD) via fiber optic cable. The APDs are connected to a computer cart which will allow a user to manage the count data formed by the APD. The Lab View program in the computer will relay information including the counts by each APD, the coincidence counts, and their errors. With this setup we can experiment with quantum entanglement.

3.2 Procedure

3.2.1 Prerequisite Steps

These steps are needed before engaging in an experiment:

1. Build setup as shown in Figure 1
2. Plug in laser diode, and APDs
3. Unlatch safety blocker from laser output
4. Start laser source using laser control program
5. Open LabView program to receive count data from APDs

3.2.2 Fringe Visibility

This experiment calls for manipulating the angle of the polarizers that collect the signal and idler photons.

![Fig. 4](image-url)
To perform the experiment we run through the prerequisite steps and then take data by holding polarizer A at a constant angle of 135 degrees while rotating polarizer B in 10 degree increments. At every increment a 20 average measurement of the coincidence count is made. We then hold polarizer B at a constant angle of 45 degrees while moving polarizer A in 10 degree increments and data is taken at each increment.

The Fringe Visibility was calculated using Eq. 1.

$$ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} $$

Eq. 1. This formula is how the calculation of the fringe visibility is made, the’ I’ denotes the amount of coincidence counts received [7]

3.2.3 Quartz Plate Manipulation

This experiment was conducted to see how the orientation of the quartz plate affected the coincidence count and which orientation will allow for a maximum of the coincident count.

The quartz plate is housed by the two mounts shown in Fig. 6. The mount on the left is used to rotate the quartz plate on a vertical axis and the mount on the right allows rotation of the quartz plate about the
optical axis. Coincidence counts were taken while adjusting the orientation of the quartz plate to see which orientation gives the best count rate.

3.2.4 Bell’s Inequality Violation

This experiment’s task was to try to violate Bell’s inequality using Equation Set 1 in three different ways. Each way was similar to the Fringe Visibility section in that the experiments call for manipulating the angle of the polarizers that collect the signal and idler photons. For each angle the average of 30 measurements of the coincidence count was taken.

\[
S = E(a, b) - E(a, b') + E(a', b) + E(a', b')
\]

\[
E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha', \beta') - N(\alpha, \beta') - N(\alpha', \beta)}{N(\alpha, \beta) + N(\alpha', \beta') + N(\alpha, \beta') + N(\alpha', \beta)}
\]

Equation Set 1 – These equations represent the method for solving for S. \(N(\alpha, \beta)\) is the coincidence count of polarizer A at \(\alpha\) degrees and polarizer B at \(\beta\) degrees [4].

- The first method of trying to violate Bell’s inequality is by rotating Polarizers A and B by preset angles that are known to give the highest value of S. We rotated the polarizers by using the mounts used in Fig. 5. We then found the coincidence counts at each of those angles, plugged it into Equation Set 1 to find a value of S.

- The next experiment is similar to the procedure done before. The angles used for the polarizers are offset from the best angles by 45 degrees. We then again plugged the coincident count values to Equation Set 1.

- The final part of this experiment involved using arbitrary angles for polarizer orientation. The coincident counts for these values were also put into Equation Set 1 to calculate a value for S.

4. Results – Samantha To

4.1 Dependence on difference of polarizer angles

Since the quantum state of an entangled photon is reliably an indicator of the quantum state of another entangled photon, then, experimentally, the determination of the polarizer angle of the entangled photon would provide information for that of the other photon. In other words, the coincidence count would have a clear dependence on the difference of polarizer angles. As previously described, there are two detectors equidistance from the source that detect the coincidence count of both detectors. Fig. 7 shows how the coincidence count oscillates with the varying polarizer angle when one polarizer is kept constant at 45 degrees and 135 degrees respectively. When we account for this offset, we can see that the maximum coincidence count occurs when the difference of polarizer angles equals zero degrees. Conversely, the minimum coincidence count occurs when the difference of polarizer angles equals 90 degrees.
Fig. 7. The coincidence count is shown with respects to the rotation of one polarizer angle while the other polarizer is kept constant at 135 degrees and 45 degrees respectively. The laser power was 20 mWatts and the acquisition time was 10 sec with 10 measurements.

Table 1 shows the single counts for both A and B detectors, the calculated accidental coincidence count, the average coincidence count as determined by the program and the net coincidence count. The accidental coincidence count was calculated with the following equation.

$$\text{Accidental Coincidence} = \frac{\text{Single Count A} \times \text{Single Count B} \times \text{Time Window (26 ns)}}{\text{Acquisition Time}}$$

The net coincidence count was obtained by subtracted the accidental coincidence count from the average coincidence count. The Fringe Visibility of both data sets were obtained with the following equation.

$$V = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}}$$

When polarizer A was kept constant at 135 degrees, the visibility was calculated to be 0.985086. Similarly, the visibility for when polarizer B was kept constant at 45 degrees was 0.983277. Both are greater than the necessary visibility value of 0.71.

<table>
<thead>
<tr>
<th>Polarizer A (Degree)</th>
<th>Polarizer B (Degree)</th>
<th>Singles A Count</th>
<th>Singles B Count</th>
<th>Accidental coincidence</th>
<th>Average coincidence</th>
<th>Net coincidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>135</td>
<td>0</td>
<td>4012.80</td>
<td>5207.25</td>
<td>1.0866</td>
<td>19.90</td>
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<td>30</td>
<td>4138.15</td>
<td>5333.10</td>
<td>1.1476</td>
<td>3.30</td>
<td>2.1524</td>
</tr>
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<td>60</td>
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<td>1.1912</td>
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<td>5629.65</td>
<td>1.2644</td>
<td>21.60</td>
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<td>120</td>
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<td>5523.65</td>
<td>1.2116</td>
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<td>5432.75</td>
<td>1.1903</td>
<td>4.35</td>
<td>3.1597</td>
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</table>
Table 1. This table shows the data set for when polarizer A is kept constant at 135 degrees. We can see numerically that the net coincidences oscillate with the minimum occurring around 30 degrees and the maximum between 120 degrees and 150 degrees. The laser power was 20mWatts and the acquisition time was 10sec with 10 measurements.

<table>
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<th>Angle</th>
<th>Delta</th>
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<th>Coeff. 2</th>
<th>Coeff. 3</th>
<th>Coeff. 4</th>
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<td>360</td>
<td>4235.45</td>
<td>5468.10</td>
<td>1.2043</td>
<td>20.25</td>
</tr>
</tbody>
</table>

Fig. 8 shows the single count for each detector when polarizer A was constant at 135 degrees and polarizer B was constant at 45 degrees. This shows the consistency of the two detectors. Clearly the detectors are consistent with themselves in both data sets, but detector B has a consistently higher single count. We note that both detectors indicate a spike in counts when the polarizer A was 135 degrees and polarizer B was 300 degrees. This spike is most likely due to an external dim light that was introduced during this data run.

4.2 Dependence on quartz plate orientation

The quartz plate is responsible for altering the polarization or phase shift of the incoming photons according to the birefringement material. To find the orientation of the quartz plate that yields the most consistent of coincidence counts, eight sets of data were obtained rotating the quartz plate about its horizontal and vertical axis. Fig. 9 shows the change in coincidence count as the quartz plate rotated about the horizontal axis. Similarly, Fig. 10 shows the change in coincidence count as the quartz plate rotated about the vertical axis. The four data sets for each Horizontal and Vertical manipulation were obtained when the polarizer angles at each detector were equal at 0,0; 45,45; 90,90 and 135,135 degrees, respectively.
This section of the experiment sought to find the orientation of the quartz plate that is independent of the polarizer angle. These points are where the four data sets intersect the most. With respects to the Horizontal axis, the quartz plate should be at an angle $2 \pm 1$ degree. With respects to the Vertical axis, the quartz plate should be at an angle $1 \pm 1$ degree.

4.3 Dependence on cosine squared of difference of polarizer angles

From the bra-ket notation of the quantum state of the entangled photons

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|V\rangle|V\rangle + |H\rangle|H\rangle)$$
we can calculate the probability of measuring the entangled photons as vertically polarized photons with the following equation.

\[ P_{VV} = \frac{1}{2} \cos^2(a - b) \]

Here we see a cosine squared relation of the difference of polarizer angles to the probability function of obtaining two vertically polarized entangled photons. This is shown in Fig. 11 and Fig. 12 with two sets of data where polarizer A is kept constant at 45 and 135 degrees and polarizer A is kept constant at 90 and 0 degrees, respectively. The fringe visibility for when the polarizer A is at 0, 45, 90 and 135 degrees was calculated to be 0.9501, 0.9852, 0.9317 and 0.9676, respectively.

Fig. 11. Coincidence count varying with a cosine square trend with respects to the polarizer angle while one polarizer is kept constant. In two data sets, the polarizer A is constant at 45 and 135 degrees respectively. The laser power was 25mWatts and the acquisition time was 0.5 sec with 20 measurements.
Fig. 12. Coincidence count varying with a cosine square trend with respects to the polarizer angle while one polarizer is kept constant. In two data sets, the polarizer A is constant at 0 and 90 degrees respectively. The laser power was 25 mWatts and the acquisition time was 0.5 sec with 20 measurements.

4.4 Violation of Bell’s Inequality

Since the entanglement of particles indicate that the quantum state of each particles are related, it would violate the Bell’s Inequality that treats each particle as an individual, unrelated to each other. This is shown by calculated the S value after obtaining a set of data as shown in Table 2. The S value is calculated with the following equation.

\[ S = |E(a, b) - E(a', b')| + |E(a', b) + E(a, b')| \]

where

\[ E(a, b) = \frac{N(a, b) + N(a, b') - N(a, b) - N(a, b')}{N(a, b) + N(a, b') + N(a, b) + N(a, b')} \]

And \( N(a, \beta) \) is the coincidence count when the polarizer angles are \( a \) and \( \beta \). The error in the S value can be calculated using error propagation.

\[ \sigma_S^2 = \sum_i^4 \left( \frac{dS}{dN_i} \sigma_i \right)^2 \]

Where \( \frac{dS}{dN_i} = \frac{dS}{dE(dN_i)} \) and \( \frac{dE}{dN_1} = \frac{2(N_3 + N_4)}{(\Sigma N_i)^2} \) and \( \frac{dE}{dN_2} = \frac{dE}{dN_3} = \frac{dE}{dN_4} = -\frac{2(N_1 + N_2)}{(\Sigma N_i)^2} \).

<table>
<thead>
<tr>
<th>Polarizer A (Degree)</th>
<th>Polarizer B (Degree)</th>
<th>Singles A Count</th>
<th>Singles B Count</th>
<th>Accidental coincidence</th>
<th>Average coincidence</th>
<th>Net coincidence</th>
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Table 2. This table shows the data set for violating the Bell’s Inequality. We note that while polarizer A is kept constant, polarizer B must have a range of 90 degrees at least to successfully calculate the Bell’s Inequality. The laser power was 25mWatts and the acquisition time was 0.5 sec with 30 measurements.

With the above calculations, the S value of one set of data was 2.518 ±0.111 which violated Bell’s Inequality by 4.666 standard deviations. A second data set was obtained and its S value was calculated to be 2.547± 0.112. Similarly, this data set also violated Bell’s Inequality by 4.578 standard deviations. A third set of data was obtained, but since the range of the mobile polarizer angle did not reach 90 degrees, the S value could not be obtained. Since both calculated S values were greater than 2 given the error, we have successfully violated Bell’s Inequality in the experiment by obtaining entangled photons.

5. Conclusion

In this experiment, we produced entangled photons and studied its properties. The maximum coincidence count of entangled photons was produced when the polarizer angles at each detector are equal. To achieve independence of polarizer angle, the quartz plate must be oriented in 2 ±1 degree about the horizontal axis and 1 ±1 degree about the vertical axis. We have experimentally shown that the probability function that the entangled photons are vertically polarized is proportional to the cosine square of the difference between polarizer angles. Finally, we proved the existence of entanglement by violating Bell’s Inequality in two sets of data by 4.666 and 4.578 standard deviations, respectively. The S value of each data runs were 2.518 ±0.111 and 2.547± 0.112 which are both greater than 2.

References


