Entanglement and Bell's Inequalities

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Abstract

Our goal for this experiment was to demonstrate violation of Bell's inequalties. This was accomplished by producing two polarization entangled photons via spontaneous parametric down-conversion using two type 1 beta barium borate crystals. With polarizers placed in front of two avalanche photodiodes, we observed how the alignment of the polarizers affected the number of observed photon pairs coincident on the photodiodes. The data obtained in this way was used to construct the S value described by the Clauser-Horne-Shimony-Holt inequality. This inequality states that S is no greater than 2, but we observed a value of S = 2.64. Thus, we demonstrated violation of a Bell's inequality, refuting the family of local realist theories of quantum mechanics.

1 Background

In quantum theory, two particles are considered entangled if their state is not separable: $|\Psi_{12}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle$. The state of an entangled particle cannot be specified without reference to the other particle. By the Copenhagen interpretation of quantum mechanics, when the state of one entangled particle is measured, the wave function collapses and the state of the other particle is also determined. Such a collapse is supposedly instantaneous, and this idea of a non-local effect historically led to objections from detractors who preferred that quantum mechanics be described by a more classical local hidden variable theory.

To illustrate the difference in these view points, consider the case of Bertlmann and his socks, as posed by Bell. Bertlmann always wears different colored socks. If you observe that he has a pink sock on one foot, you know the other sock he is wearing is not pink, and this information is gained instantly without needing to observe the other sock.

Contrast Bertlmann's socks with the case of entangled particles. In quantum mechanics, entangled particles are treated as being in a superposition of possible states. We cannot be certain of what state we will measure a particle as being in, and by the Copenhagen interpretation, we cannot even speak meaningfully of a definite state measurement result because the state of each particle is undetermined prior to observation. Upon observation the state of both particles collapse instantly to a single state. This differs from the case of Bertlmann in two crucial ways: 1) Bertlmann was wearing a pink sock regardless of whether you observed his socks or not and 2) the observation of Bertlmann wearing one pink sock did not affect the color of the other sock[1]. In particular, the concept that an observation of one particle state could instantly alter the state of another particle regardless of spatial separation provides the basis for the objection to quantum mechanical formalism given by Einstein, Poldolsky, and Rosen[2], who disputed the existence of such a "spooky action at a distance"[3].

Bell's inequalities are significant in that they allow experimental testing of the opposing quantum and classical interpretations. A Bell's inequality is simply a mathematical inequality derived by assuming locality and conterfactual definiteness, conterfactual definiteness referring to the realist view that the state of particles are determined by some hidden variables that give a measurement of the state a definite result. Bell's inequalities can be calculated for classical quantities, but inequalities for classical examples are obviously upheld and therefore not of great interest. Such inequalities suddenly become interesting when considering quantum cases, where it is predicted and observed that these inequalities are violated[4]. Observed violation of Bell's inequalities leads to the conclusion that a local realist theory cannot be used to reproduce all of the predictions of quantum mechanics, as stated by Bell's theorem[5].

2 Theory

The Bell's inequality relevant to this experiment is the Clauser-Horne-Shimony-Holt(CHSH) inequality[6]. First, consider two polarization entangled particles, in the state $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i)$, where V and H refer to vertical and horizontal configurations and s and i are the historical way to label photons produced by spontaneous parametric down-conversion. This state is invariant regardless of choice of polarization basis. To show this, consider a rotation by an angle α, so we will make use of the rotated basis states of $|V_\alpha\rangle = \cos(\alpha)|V\rangle + \sin(\alpha)|H\rangle$ and $|H_\alpha\rangle = -\sin(\alpha)|V\rangle + \cos(\alpha)|H\rangle$. In this basis,

$$\begin{aligned}
\frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i) &= \frac{1}{\sqrt{2}}[(\cos(\alpha)|V\rangle_s + \sin(\alpha)|H\rangle_s)(\cos(\alpha)|V\rangle_i + \sin(\alpha)|H\rangle_i) \\
&+ (-\sin(\alpha)|V\rangle_s + \cos(\alpha)|H\rangle_s)(-\sin(\alpha)|V\rangle_i + \cos(\alpha)|H\rangle_i)] \\
&= \frac{1}{\sqrt{2}}[\cos^2(\alpha)|V\rangle_s|V\rangle_i + \sin^2(\alpha)|H\rangle_s|H\rangle_i \\
&+ \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i + \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i \\
&+ \sin^2(\alpha)|V\rangle_s|V\rangle_i + \cos^2(\alpha)|H\rangle_s|H\rangle_i \\
&- \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i - \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i] \\
&= \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i) \\
&= |\Psi_{Bell}\rangle
\end{aligned}$$

so $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i)$. Thus, $|\Psi_{Bell}\rangle$ is invariant under rotation.

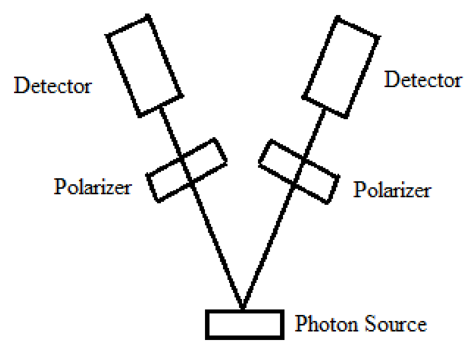


Figure 1: Outline of the thought experiment. Polarization entangled photon pairs are emitted from a source, pass through polarizers and are received by a detector.

Consider now measuring the polarization of these photons. As outlined in Figure 1, imagine situating two detectors so that each detector will receive one of the entangled photons, and then imagine placing a polarizer in front of each detector, one polarizer rotated to the arbitrary angle α, the other to the angle β. The probability that both photons are vertical in the bases of their respective polarizers is

$$
\begin{aligned}
P_{VV}(\alpha,\beta) &= |\langle V_\alpha|_i \langle V_\beta|_s |\Psi_{Bell}\rangle|^2 \\
&= \frac{1}{2}|[\langle V|_i \cos(\alpha) + \langle H|_i \sin(\alpha)][\langle V|_s \cos(\beta) + \langle H|_s \sin(\beta)][|V\rangle_i|V\rangle_s + |H\rangle_i|H\rangle_s]|^2 \\
&= \frac{1}{2}|\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)|^2 \\
&= \frac{1}{2}\cos^2(\alpha-\beta).
\end{aligned}
$$

Similarly, the probabilities for other polarization measurements are

$$
\begin{aligned}
P_{HH}(\alpha,\beta) &= \frac{1}{2}\cos^2(\alpha-\beta) \\
P_{VH}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta) \\
P_{HV}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta).
\end{aligned}
$$

Defining $N(\alpha,\beta)$ as the number of coincident photon counts when the polarizers are aligned at angles α and β, it is obvious that

$$
\begin{aligned}
P_{VV}(\alpha,\beta) &= \frac{N(\alpha,\beta)}{N_{Total}} \\
P_{HH}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta_\perp)}{N_{Total}} \\
P_{VH}(\alpha,\beta) &= \frac{N(\alpha,\beta_\perp)}{N_{Total}} \\
P_{HV}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta)}{N_{Total}},
\end{aligned}
$$

where $\alpha_\perp = \alpha + 90°$ and $\beta_\perp = \beta + 90°$. N_{Total} refers to the total number of coincident pairs detected: $N_{Total} = N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)$. Knowing this, it is easy to measure the probabilities P_{VV}, P_{HH}, etc. by simply measuring the number of coincident photon pairs detected when the polarizers are aligned appropriately.

Consider now the correlation function

$$
\begin{aligned}
E(\alpha,\beta) &= P_{VV} + P_{HH} - P_{VH} - P_{HV} \\
&= \frac{1}{2}\cos^2(\alpha-\beta) + \frac{1}{2}\cos^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) \\
&= \cos^2(\alpha-\beta) - \sin^2(\alpha-\beta) \\
&= \cos(2(\alpha-\beta)).
\end{aligned}
$$

With this quantity in mind, we can make use of the Clauser-Horne-Shimony-Holt inequality.

The Clauser-Horne-Shimony-Holt (CHSH) inequality concerns the quantity S defined as

$$S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')|.$$

Specifically, the CHSH inequality states that

$$|S| \leq 2.$$

If, however, $a = -\pi/4$, $a' = 0$, $b = -\pi/8$, $b' = \pi/8$, then $S = |-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}| + |\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}| = 2\sqrt{2} > 2$. Thus, quantum mechanics predicts that the CHSH inequality can be violated for certain polarizer alignments.

3 Experimental Setup

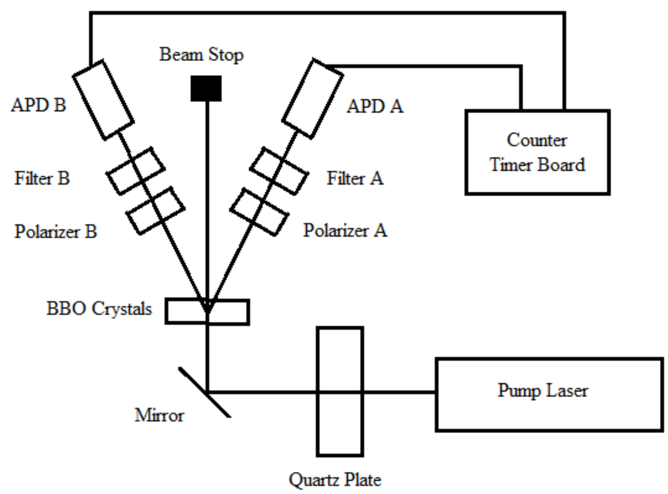


Figure 2: The pump laser drives the process of spontaneous parametric down-conversion in the BBO crystals, which causes the crystals to occaisonally emit polarization entangled photons. These down-converted photons are received by single-photon counting avalanche photodiodes (APDs). Polarizers are placed in front of the photodiodes to allow selection of the polarization of photons incident on the APDs. A quartz plate is used to correct for phase differences between different polarizations of down-converted photons. Filters are also placed in front of the APDs, which are meant to only admit wavelengths matching the wavelength of the down-converted photons. The APDs send out a TTL pulse for every incident photon. The pulses are recieved by a counter time board, which has a time window of 26 ns for counting coincident pulses.

The apparatus used in this experiment consists of a argon ion laser, two type 1 beta barium borate (BBO) crystals, a quartz plate, a pair of single-photon counting avalanche photodiodes (APDs), and a pair of polarizers. An outline of the apparatus is shown in Figure 2. The laser is used to pump the BBO crystals, which produce polarization entangled photons due to spontaneous parametric down-conversion (SPDC). These photon pairs are received by the APDs and polarizers are placed in front of each APD to filter the polarization of photons incident on the APDs[7].

Entangled photons are produced via spontaneous parametric down-conversion (SPDC). This process refers to the emission of entangled photon pairs by nonlinear crystals. From historical precedent, one of these photons is called the "signal" photon, and the other is called the "idler" photon. When the pump laser hits the BBO crystals, there is a small chance that a photon is absorbed and then reemitted as two photons. The crystal is unaltered by this process, so energy and momentum are conserved, as illustrated in figure 3, and the wavelengths of the down-converted photons emitted by the BBO crystals are double the wavelength of the pump laser.

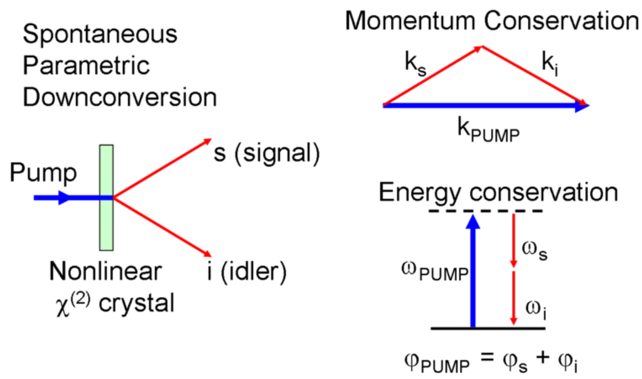


Figure 3: A diagram illustrating the aspects of spontaneous parametric down-conversion (SPDC). Two photons are emitted: the signal photon and the idler photon. Conservation of momentum and energy relate the frequency and wave numbers of the down-converted photons to the absorbed photon.

Type 1 BBO crystals emit photons pairs that are polarization entangled: the signal photon will have the same polarization as the idler photon. The pump laser is linearly polarized and the two BBO crystals are aligned perpendicular to each other and 45° relative to the pump laser so that SPDC occurs with both crystals. One crystal will emit a pair of photons vertically polarized while the other emits photon pairs that are both horizontally polarized. Thus, a photon pair emitted from the crystals will have an equal chance to be either both horizontally polarized or both vertically polarized. This corresponds to a state of the (unnormalized) form $|\Psi\rangle = |V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i$, which is the state desired to test for violation of the CHSH inequality.

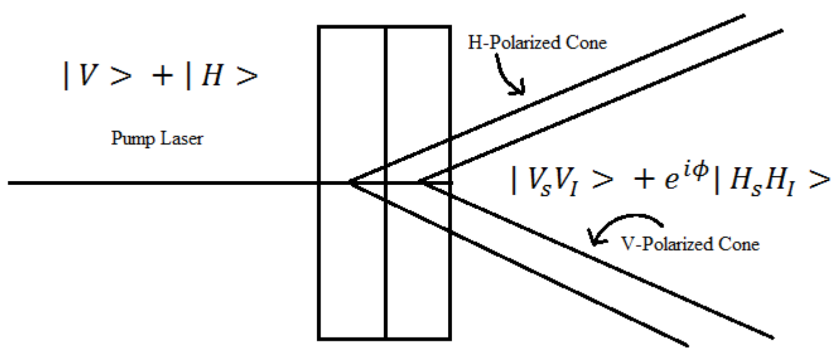


Figure 4: The pair of BBO crystals absorbs photons and then emits polarization entangled photons outwards in a cone. The crystals are spaced apart, so down-converted photons emitted from one crystal travel a different distance than photons emitted from the other, introducing a phase shift factor into the state of the down-converted photons.

The two BBO crystals are spatially separated by a small distance, as shown in Figure 4, so there is a phase difference between the different polarizations of photon pairs. This would give the state $|\Psi\rangle = |V\rangle_s|V\rangle_i + e^{i\phi}|H\rangle_s|H\rangle_i$. To correct for this phase factor, a quartz plate is used to introduce a phase shift in the horizontal polarization of the pump laser.

The BBO crystals emit the down-converted photons outwards in a cone shape. The two APDs are situated to detect photons on opposite ends of the circumference of this cone so that any entangled photons will be incident on both APDs at the same time. When photons are incident on the APDs, the APDs send out a TTL pulse. These TTL pulses are received by a counter timer board. A LabView program records the results of the counter timer board and displays how many photons were incident on each APD in a given time period. The program also counts the number of times photons are detected coincidentally, that is when the APDs detect an incident photon at the same time. The counter timer board actually records photons as coincident if the TTL pulses are within 26 ns of each other, so some coincident counts do not correspond to entangled photons. Correcting for these accidental coincident detections, the coincident count should be equal to the number of entangled photon pairs received in both APDs.

Polarizers are placed in front of both APDs. This allows for filtering of the polarization of photons incident on the APDs and therefore testing of the CHSH inequality. First, however, the entanglement of the photons must be tested. This is done by holding the alignment angle of one polarizer constant while altering the alignment of the other polarizer. For entangled photons, the coincident count should display a cosine squared dependence on the difference in polarizer angles. After this is confirmed, violation of the CHSH inequality can be tested using, for example, the polarizer angles mentioned in the theory section.

4 Procedure and Results

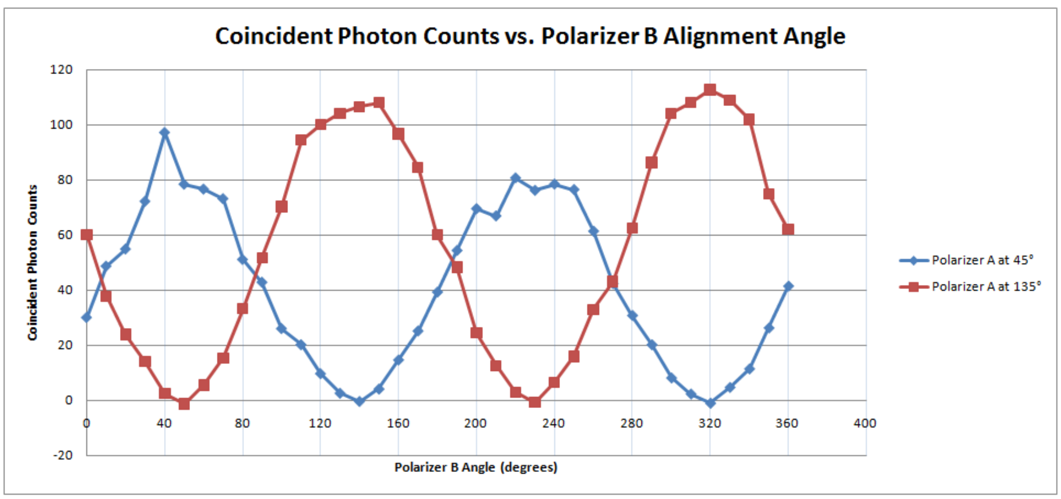


Figure 5: Plot of coincident counts as the angle of polarizer B is altered. Data was taken both when polarizer A was aligned at 45° and at 135°. Results are roughly sinusoidal, with the expected phase shift of 90° between the two different alignments of A.

We did not immediately try to violate the CHSH inequality. Instead, the initial tests done using the apparatus were simpler tests to see whether such violation was possible. Our first experiment was to check for cosine squared dependence on the difference of polarizer angles. This was accomplished by keeping one polarizer at a constant angle and measuring the coincident counts on the APDs while altering the angle of the other polarizer. This cosine squared dependence test was done to confirm that the setup was aligned well enough that observing violation of Bell's inequalities might be possible. The results of this preliminary test are displayed in figure 5 and figure 6. The sinusoidal appearance of the data in figure 5 suggested that observing violation of Bell's inequality might indeed be possible, alhough minor sinusoidal alteration in the single counts, as shown in figure 6. suggests that the down-converted light was not perfectly unpolarized. Subsequently attempting to violate the CHSH inequality did not yield an S value greater than 2. Thus, improving the alignment of the apparatus was required.

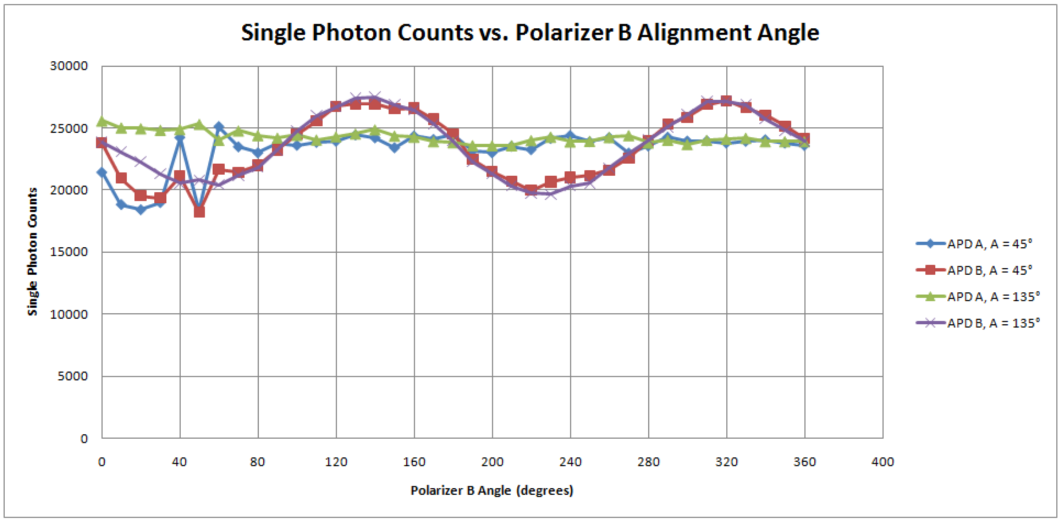


Figure 6: Plot of single photons counts in APD A and APD B as the angle of polarizer B is altered. Data was taken both when polarizer A was aligned at 45° and at 135°. Results are roughly linear, but with some sinusoidal alteration in counts observed.

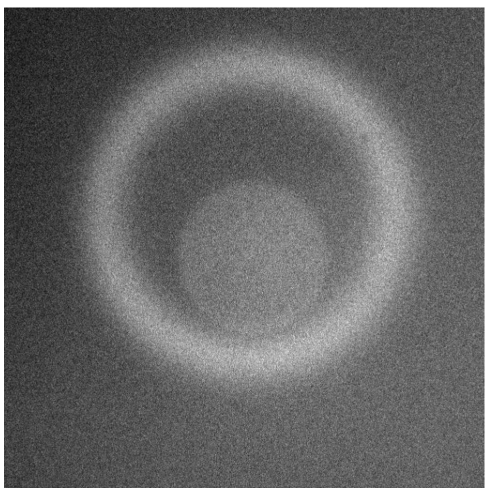
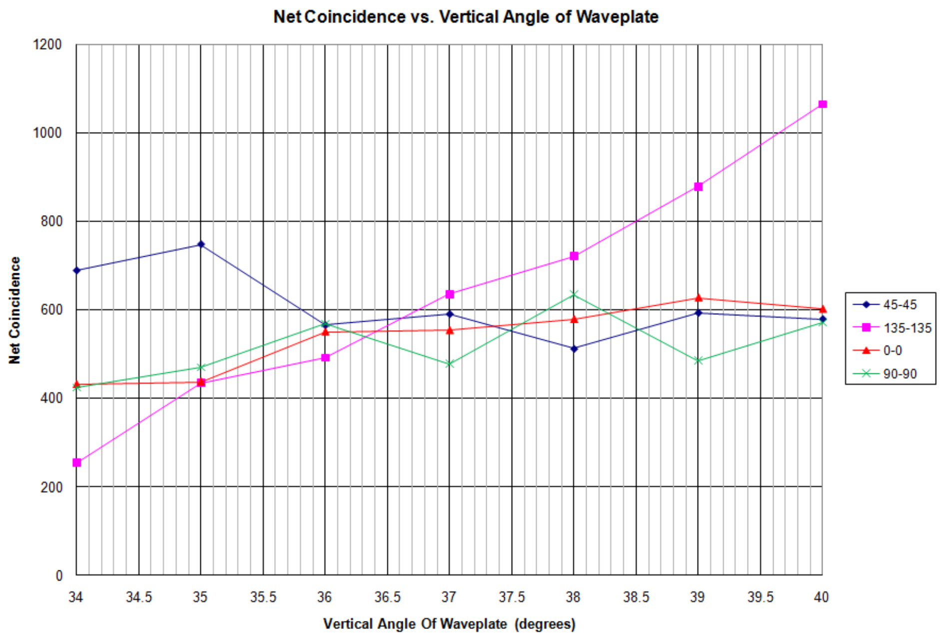
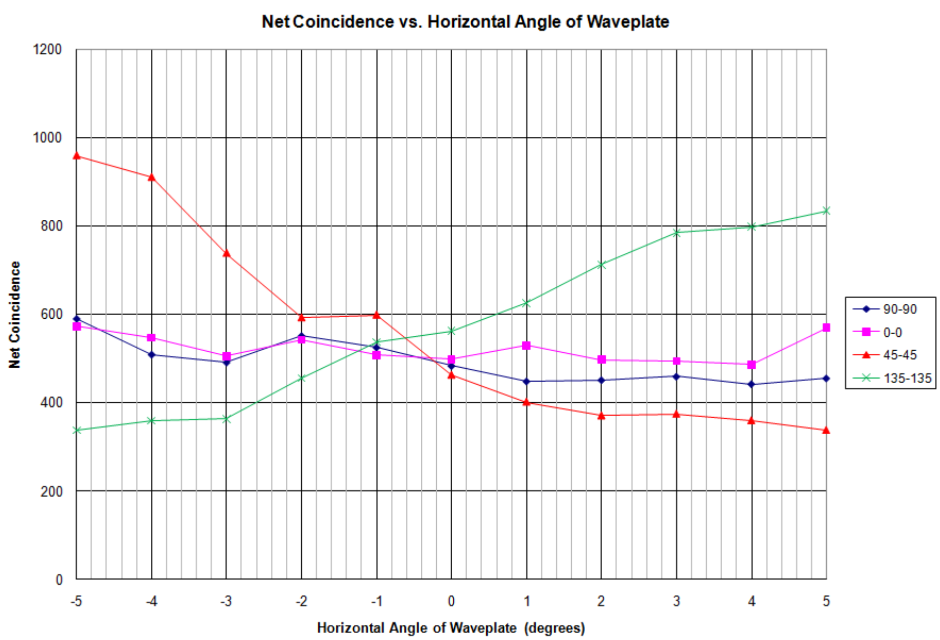


Figure 7: A CCD camera was used to image the ring produced by spontaneous parametric down-conversion. The circle in the center of the ring is an artifact caused by unfiltered fluorescence from the pump laser's argon ion tube.

Much time was spent realigning the BBO crystals and the quartz plate. To align the BBO crystal, we used a CCD camera instead of APDs to image the ring of photons emitted via SPDC. Satisfactory alignment of the BBO crystals was reached when a single clear ring was observed (as shown in figure 7) and the light was tested to be apparently unpolarized.



(a) Vertical Alignment



(b) Horizontal Alignment

Figure 8: An example of the data taken to align the waveplate, (a) referring to vertical alignment and (b) referring to horizontal alignment. This is a plot of the number of observed photon pairs coincident on the APDs for different vertical waveplate alignments. Counts were acquired over a 5 second acquisition period. We used four different polarizers configurations: both polarizers aligned at 0°, both at 45°, both at 90°, and both at 135°. We kept the vertical alignment at 36.5° and horizontal alignment at 359.5°, as the coincident counts are roughly equal for different polarizer angles given this alignment.

The waveplate was aligned by keeping both polarizers parallel while altering the alignment of the polarizers and waveplate, an example of which is given in figure 8. We chose to vertically align the waveplate to 36.5° and to horizontally align the waveplate to 359.5°, as these were the alignment angles that yielded similar coincident counts regardless of polarizer angles.

After performing these alignment procedures, we again tested for violation of the CHSH inequality. This was accomplished by finding the coincident counts needed to construct the correlation functions comprising S. We needed to calculate four correlation functions, each of which requires coincident count measurements at four different polarizer angle configurations. Thus, we used the APDs with a 5 second acquistion to measure photon counts for sixteen different polarizer configurations, the results of which are displayed on table 1. We also measured the total number of photons incident on the APDs during this acquistion time so that we could correct for unentangled photons accidentally being detected as coincident photons. Accidental coincident counts are corrected for using the formula:

$$\text{Net Coincidence} = \frac{(\text{Single Count A})(\text{Single Count B})(\text{Acquisition Time} = 5\text{ s})}{\text{Time Window of Counter Board} = 26\text{ ns}}.$$

A record of the raw data found, including single counts and the calculated accidental counts, is given in the appendix.

Angle Of Polarization, A	Angle Of Polarization, B	Average Coincidence	Net Coincidence	Net Standard Deviation
0°	22.5°	351	344	39.71
0°	67.5°	102	95	2.60
0°	112.5°	74	66	16.63
0°	157.5°	309	302	2.99
45°	22.5°	382	373	18.57
45°	67.5°	328	321	19.64
45°	112.5°	80	73	8.02
45°	157.5°	46	38	4.76
90°	22.5°	89	80	8.71
90°	67.5°	385	375	10.16
90°	112.5°	407	398	15.97
90°	157.5°	127	117	6.50
135°	22.5°	106	96	4.96
135°	67.5°	52	42	3.28
135°	112.5°	391	381	23.37
135°	157.5°	465	455	11.51

Table 1: Average coincident counts found for various polarizer configurations. The net coincident counts are the average coincident counts corrected for accidental coincidences. The standard deviation in the net count is also included. The polarizer angles were chosen to try to maximize S.

Using the net coincident counts from table 1, constructing the correlation functions and finding S is simply a matter of arithmetic. Error was calculated from the net coincidence count standard deviations using typical propagation of uncertainty techniques. The values found for the correlation functions are

$$E(0°, 22.5°) = 0.67 \pm 0.04,$$
$$E(0°, 157.5°) = 0.52 \pm 0.01$$
$$E(135°, 22.5°) = -0.63 \pm 0.02$$
$$E(135°, 157.5°) = 0.81 \pm 0.09$$

which corresponds to an S value of

$$S = 2.64 \pm 0.10.$$

Our S value violates the CHSH inequality.

The predicted value for S was $2\sqrt{2}$, which corresponds to a relative error of 0.0666. The fact that we did not observe the theoretically predicted S value is indicative of some small imperfections in the alignment of the apparatus. Nonetheless, violation of a Bell's inequality is clearly demonstrated.

We also measured S using arbitrarily chosen polarizer angles (while still holding to the conditions needed to calculate S). The average coincident counts for this second test are displayed in table 2.

Angle Of Polarization, A	Angle Of Polarization, B	Average Coincidence	Net Coincidence	Net Standard Deviation
0°	32°	294	287	14.50
0°	90°	45	37	5.63
0°	122°	121	113	11.26
0°	180°	419	411	23.34
60°	32°	389	380	15.47
60°	90°	366	357	10.30
60°	122°	132	123	13.83
60°	180°	134	126	8.07
90°	32°	143	134	1.59
90°	90°	428	420	11.37
90°	122°	348	340	13.62
90°	180°	52	43	6.90
150°	32°	149	141	8.68
150°	90°	125	117	3.27
150°	122°	362	354	17.80
150°	180°	396	387	32.16

Table 2: The second set of average coincident counts found for various polarizer configurations. The net coincident counts are the average coincident counts corrected for accidental coincidences. The standard deviation in the net count is also included. The polarizer angles were chosed on a whim, rather than to try and maximize S.

These arbitrarily chosen angles yield correlation functions

$$E(0^\circ, 32^\circ) = 0.43 \pm 0.02,$$
$$E(0^\circ, 90^\circ) = 0.82 \pm 0.02$$
$$E(60^\circ, 32^\circ) = -0.47 \pm 0.03$$
$$E(60^\circ, 90^\circ) = 0.51 \pm 0.18$$

corresponding to an S value of

$$S = 2.24 \pm 0.18.$$

Although the polarizer angles were chosen arbitrarily, this S value still violates the CHSH inequalitiy. The calculated theoretical S value was found to be $S = 2.497$, so the value of 2.24 is not surprising. The relative error for this S value is 0.103, which is slightly higher than the previous S value but still acceptable, and still demonstrates violation of Bell's inequalities for other angle configurations.

5 Conclusion

In conclusion, we used a pair of type 1 beta barium borate crystals to produce polarization entangled photons through spontaneous parametric down-conversion. We utilized a pair of polarizers and avalanche photodiodes to detect these polarization entangled photon pairs. After careful alignment, we were able to demonstrate violation of the Clauser-Horne-Shimony-Holt inequality at theoretically predicted polarizer alignments. This violation of a Bell's inequality demonstrates the entanglement of the photons and supports Bell's theorem, which states that no system based on local hidden variables can explain all of the phenomena of quantum mechanics.

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Appendix

Below are the raw data taken for various polarizations angle configurations, giving the measured photons counts into a single polarizer, the coincident counts, the calculated accidental coincident counts, and the net counts (average counts corrected for accidental counts).

First we used polarizer angles $a = 45°$, $a' = 0°$, $b = 67.5°$, $b' = 22.5°$.

First trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	22.5°	41813	32753	384	7	377
0°	67.5°	41717	33007	100	7	93
0°	112.5°	41597	33288	55	7	48
0°	157.5°	41708	33101	312	7	305
45°	22.5°	4646	36537	403	9	394
45°	67.5°	41238	32781	309	7	302
45°	112.5°	41459	33505	71	7	64
45°	157.5°	41946	33293	42	7	35
90°	22.5°	46391	37309	99	9	90
90°	67.5°	46115	37632	390	9	381
90°	112.5°	47849	38984	423	10	413
90°	157.5°	46668	38168	134	9	125
135°	22.5°	45759	37576	111	9	102
135°	67.5°	48317	39571	51	10	41
135°	112.5°	47410	39216	372	10	362
135°	157.5°	46982	38442	453	9	444

Second trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	22.5°	41362	32378	307	7	300
0°	67.5°	41398	32442	105	7	98
0°	112.5°	41146	33160	79	7	72
0°	157.5°	41869	33566	310	7	303
45°	22.5°	46932	36412	373	9	364
45°	67.5°	40764	32016	348	7	341
45°	112.5°	41175	32917	86	7	79
45°	157.5°	41788	33252	44	7	37
90°	22.5°	45764	36720	87	9	78
90°	67.5°	47504	38383	391	9	382
90°	112.5°	47731	39452	408	10	398
90°	157.5°	47658	38210	124	9	115
135°	22.5°	47443	38686	103	10	93
135°	67.5°	48288	39508	50	10	40
135°	112.5°	47706	39494	417	10	407
135°	157.5°	47506	38012	476	9	467

Third trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	22.5°	42054	32658	363	7	356
0°	67.5°	41229	32519	102	7	95
0°	112.5°	41895	33462	87	7	80
0°	157.5°	41445	32906	306	7	299
45°	22.5°	46639	36135	369	9	360
45°	67.5°	40837	32194	327	7	320
45°	112.5°	41442	33039	83	7	76
45°	157.5°	41764	33142	51	7	44
90°	22.5°	46416	37333	82	9	73
90°	67.5°	46905	38308	373	9	364
90°	112.5°	47127	39266	391	10	381
90°	157.5°	47053	38215	122	9	113
135°	22.5°	47465	38623	103	10	93
135°	67.5°	47966	39372	56	10	46
135°	112.5°	47234	38993	383	10	373
135°	157.5°	46853	38204	465	9	456

Then we used polarizer angles $a = 0°$, $a' = 60°$, $b = 32°$, $b' = 90°$.

First trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	32°	40592	31603	277	7	270
0°	90°	41867	34348	50	7	43
0°	122°	43284	36843	108	8	100
0°	180°	44613	36422	394	8	386
60°	32°	45579	35765	400	8	392
60°	90°	46383	37212	369	9	360
60°	122°	46703	38279	139	9	130
60°	180°	45262	36148	125	9	116
90°	32°	44824	35269	143	8	135
90°	90°	45213	36748	425	9	416
90°	122°	45391	36800	362	9	353
90°	180°	45642	36531	56	9	47
150°	32°	45404	35660	139	8	131
150°	90°	45854	36589	123	9	114
150°	122°	45552	37742	383	9	374
150°	180°	45381	36491	417	9	408

Second trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	32°	40940	32336	302	7	295
0°	90°	42273	34678	39	8	31
0°	122°	43828	37070	126	8	118
0°	180°	44591	36570	440	8	432
60°	32°	45418	35780	371	8	363
60°	90°	46613	37234	375	9	366
60°	122°	46304	38325	141	9	132
60°	180°	45486	36935	138	9	129
90°	32°	44896	35550	141	8	133
90°	90°	45277	36524	419	9	410
90°	122°	45361	37016	348	9	339
90°	180°	45284	36524	44	9	35
150°	32°	46083	35523	153	9	144
150°	90°	45637	36745	124	9	115
150°	122°	45654	37030	352	9	343
150°	180°	45621	36316	359	9	350

Third trial results:

Angle Of Polarization, A	Angle Of Polarization, B	Single Count, A	Single Count, B	Coincidence Count	Accidental Count	Net Count
0°	32°	42050	33085	303	7	296
0°	90°	42430	34831	46	8	38
0°	122°	44101	36927	129	8	121
0°	180°	44245	36850	424	8	416
60°	32°	45932	35938	395	9	386
60°	90°	46418	37626	355	9	346
60°	122°	46456	37856	116	9	107
60°	180°	45000	36629	140	9	131
90°	32°	44910	35009	144	8	136
90°	90°	45143	36705	441	9	432
90°	122°	45359	37836	335	9	326
90°	180°	45029	36918	56	9	47
150°	32°	46005	35433	155	8	147
150°	90°	45093	36769	129	9	120
150°	122°	45360	37050	352	9	343
150°	180°	45174	36341	412	9	403