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Ghost Imaging: From Quantum to Classical to Computational

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Optical and Quantum Communications Group
The Truth about Ghost Imaging

- Biphoton ghost imaging
- Pseudothermal ghost imaging
- Unified Gaussian-state resolution and field-of-view analysis
- Signal-to-noise ratio behavior
- Spatial light modulator (SLM) ghost imaging
- Computational ghost imaging
- Potential for aberration immunity
- Discussion
**SPDC and the Biphoton State**

- **Spontaneous parametric downconversion (SPDC)**

  ![Diagram of SPDC process](image)

  - strong pump at frequency $\omega_P$
  - no input at signal frequency $\omega_S$ or at idler frequency $\omega_I$
  - nonlinear mixing produces signal and idler outputs that are *entangled* in frequency and momentum
    \[ \omega_P = \omega_S + \omega_I \text{ and } k_P = k_S + k_I \]
  - with type-II phase matching signal and idler are orthogonally polarized
  - at low flux these entangled outputs form a stream of biphons
Ghost Imaging in the Biphoton Limit

- Pittman *et al*. ghost imaged a transmission mask
  - used biphoton-state source and coincidence counting
  - it’s a ghost image because the bucket detector has no spatial resolution and the object is not in the path of the pinhole detector
  - attributed this behavior to entanglement of signal and idler photons

Pittman *et al.*
*Phys Rev A* 1995
**Pseudothermal Ghost Imaging**

- **Scarcelli et al.** ghost imaged a transmission mask
  - used pseudothermal light and photocurrent correlation
  - it’s a ghost image because the bucket detector has no spatial resolution and the object is not in the path of the pinhole detector
  - attributed this behavior to nonlocal two-photon interference

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**Image - Near Field**

**Object**

**Image**

**Scarcelli et al.**

*Phys Rev Lett 2006*
The Toy Soldier

- Meyers *et al.* ghost imaged a toy soldier
  - used pseudo-thermal light and coincidence counting
  - it’s a ghost image because the bucket detector has no spatial resolution and the object is not in the path of the CCD array
  - attributed this behavior to nonlocal two-photon interference

Meyers *et al.*
*Phys Rev A* 2008
Classical versus Quantum Imaging

- High-sensitivity photodetection is...
  - always quantum, because light is quantum mechanical and photodetection is a quantum measurement

- High-sensitivity photodetection performance may often be...
  - calculated semiclassically, by assuming light is classical and the electron charge is discrete, so that the noise behavior is Poisson shot noise plus classical-light excess noise

- Semiclassical theory is quantitatively correct...
  - when light is in a coherent state or a mixture thereof and standard photodetection (direct, homodyne, or heterodyne) is employed

- Imaging performance is truly quantum if...
  - it cannot be explained by semiclassical theory
Unified Gaussian-State Framework

Gaussian states include...

- laser light, LED light, sunlight, i.e., “classical states”
- low-flux biphoton output from SPDC, viz., a “quantum” state

Gaussian states are...

- characterized by their mean values and coherence functions
- closed under linear transformations like free-space diffraction
Zero-Mean Gaussian States

- Positive-frequency, photon-units field operator: $\hat{E}_z(\rho, t) e^{i k_0 z - i \omega_0 t}$
  - paraxial, $z$-propagating
  - $[\hat{E}_z(\rho_1, t_1), \hat{E}_z^\dagger(\rho_2, t_2)] = \delta(\rho_2 - \rho_1) \delta(t_2 - t_1)$

- Zero-mean Gaussian state completely characterized by
  - phase-insensitive correlation function: $\langle \hat{E}_z^\dagger(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle$
  - phase-sensitive correlation function: $\langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle$

- If $\langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle = 0$
  - state is always classical (has proper $P$-representation)
  - laser light, LED light, thermal light

- If $\langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle \neq 0$
  - state may be classical or nonclassical
  - squeezed light, classical phase-sensitive light
Quantum Huygens-Fresnel Principle Propagation

- Correlation propagation from \( z = 0 \) to \( z = L \)

\[
\hat{E}_0(\rho, t) \rightarrow \hat{E}_L(\rho', t) = \int d\rho \hat{E}_0(\rho, t - L/c) \frac{\exp(i k_0 |\rho' - \rho|^2/2L)}{i \lambda_0 L h_L(\rho' - \rho)}
\]

\[
\equiv \langle \hat{E}_L(\rho'_1, t + \tau) \hat{E}_L(\rho'_2, t) \rangle
\]

\[
K^{(p)}_L(\rho'_1, \rho'_2, \tau) = \int \int d\rho_1 d\rho_2 K^{(p)}_0(\rho_1, \rho_2, \tau) h_L(\rho'_1 - \rho_1) h_L(\rho'_2 - \rho_2)
\]

\[
\equiv \langle \hat{E}_L^\dagger(\rho'_1, t + \tau) \hat{E}_L(\rho'_2, t) \rangle
\]

\[
K^{(n)}_L(\rho'_1, \rho'_2, \tau) = \int \int d\rho_1 d\rho_2 K^{(n)}_0(\rho_1, \rho_2, \tau) h_L^*(\rho'_1 - \rho_1) h_L(\rho'_2 - \rho_2)
\]
Gaussian-State Correlation Functions

- Gaussian Schell-model phase-insensitive auto-correlation

\[ \langle \hat{E}_\ell^\dagger (\rho_1, t_1) \hat{E}_\ell (\rho_2, t_2) \rangle = \frac{2P}{\pi a_0^2} e^{- (|\rho_1|^2 + |\rho_2|^2)} e^{\frac{d_0^2}{2} |\rho_2 - \rho_1|^2 / 2\rho_0^2} e^{-(t_2 - t_1)^2 / 2 T_0^2} \]

\( \ell = S, R \)  

- Thermal (and pseudothermal) light
  - phase-insensitive cross-correlation = phase-insensitive auto-correlation
  - no phase-sensitive auto-correlation or cross-correlation

- Phase-sensitive light
  - no phase-insensitive cross-correlation
  - no phase-sensitive auto-correlation
  - maximum quantum phase-sensitive cross-correlation
Biphoton Ghost Imaging

SPDC

PBS

\[ \hat{E}_R(\rho, t) \]
\[ \hat{E}_S(\rho, t) \]
\[ \hat{E}_1(\rho, t) \]

L-m free space propagation

object, \( T(\rho) \)

bucket detector (fixed)

pinhole detector, center \( \rho_1 \) (scanning)

\[ i_1(t) \]

\[ i_2(t) \]

\[ C(\rho_1) \]
Biophoton Ghost Imaging

- Assume Gaussian-Schell model source:
  photon flux $P$, intensity radius $a_0$, coherence radius $\rho_0 \ll a_0$

- Assume far-field operation: $k_0 a_0^2 / 2L \ll 1$

- Assume object lies within $\sqrt{2} \lambda_0 L / \pi \rho_0$ field of view

- Photocurrent cross-correlation function

  \[
  C(\rho_1) = q^2 \eta_1 \eta_2 A_1 \left( \frac{2P}{\pi a_L^2} \right)^2 \left[ \int_{A_2} d\rho \left| T(\rho) \right|^2 \right. \\
  + \left. \sqrt{\frac{1}{8\pi PT_0 \rho_0^2}} \int_{A_2} d\rho \ e^{-|\rho_1 + \rho|^2 / \rho_L^2} \left| T(\rho) \right|^2 \right]
  \]

  intensity radius $a_L = \lambda_0 L / \pi \rho_0$, coherence radius $\rho_L = \lambda_0 L / \pi a_0$

Erkmen & Shapiro, PRA (2008)

www.rle.mit.edu/qoptics
Pseudo-thermal Ghost Imaging

- cw laser
- rotating ground glass
- object, $T(\rho)$
- bucket detector (fixed)
- pinhole detector, center $\rho_1$ (scanning)
- correlator

Mathematical expressions and relations:

- $E(\rho, t)$
- $E_S(\rho, t)$
- $E_1(\rho, t)$
- $E_R(\rho, t)$
- $L$-m free space propagation
- $C(\rho_1)$

$i_1(t)$ and $i_2(t)$ represent the intensities detected by the pinhole and bucket detectors, respectively.
Pseudothermal Ghost Imaging

- Assume Gaussian-Schell model source: photon flux $P$, intensity radius $a_0$, coherence radius $\rho_0 \ll a_0$
- Assume far-field operation: $k_0 a_0 \rho_0 / 2L \ll 1$
- Assume object lies within $\lambda_0 L / \pi \rho_0$ field of view
- Photocurrent cross-correlation function

$$C(\rho_1) = q^2 \eta_1 \eta_2 A_1 \left( \frac{2P}{\pi a_L^2} \right)^2 \left[ \int_{A_2} d\rho \left| T(\rho) \right|^2 \right]$$

$$+ \int_{A_2} d\rho e^{-|\rho_1 - \rho|^2 / \rho_L^2} \left| T(\rho) \right|^2$$

intensity radius $a_L = \lambda_0 L / \pi \rho_0$, coherence radius $\rho_L = \lambda_0 L / \pi a_0$

Erkmen & Shapiro, *PRA* (2008)
Dual-Wavelength Operation

- Assume Gaussian-Schell model source
  - photon flux $P$, intensity radius $a_0$, coherence radius $\rho_0 \ll a_0$

- Use nondegenerate type-II SPDC
  - signal frequency $\omega_S$, idler frequency $\omega_I$

- Use unequal path lengths
  - signal path length $L_S$, idler path length $L_I$

- Assume far-field operation: $k_S a_0^2 / 2 L_S, k_I a_0^2 / 2 L_I \ll 1$

- Image is focus when $k_S / L_S = k_I / L_I$

- Spatial resolution set by $\lambda_S L_S / \pi \rho_0 = \lambda_I L_I / \pi \rho_0$
What about Signal-to-Noise Ratio?

- **Source coherence time:** $T_0$
- **Photodetector response time:** $T_d$
- **Cross-correlation integration time:** $T_I$
- **Broadband biphoton imaging:** $T_0 \ll T_d \ll T_I$
  
  \[
  \text{SNR} \rightarrow \frac{2\eta_1\eta_2 P T_I A_1}{\pi a_L^2} |T(\rho_1)|^2 \text{ with increasing } P
  \]
  
  but *only* in the biphoton limit
- **Narrowband pseudothermal imaging:** $T_d \ll T_0 \ll T_I$
  
  \[
  \text{SNR} \rightarrow \sqrt{2\pi} \frac{T_I}{T_0} \frac{\rho_L^2}{A'_T} |T(\rho_1)|^4 \text{ with increasing } P
  \]
  
  $A'_T = \text{effective area of object}$

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What about Image Acquisition Time?

- Image acquisition time ($T_I$) is the averaging time needed to achieve a target value for SNR.

- Comparison between broadband biphoton source ($T_I^{(q)}$) and narrowband pseudothermal source ($T_I^{(c)}$).

\[
\frac{T_I^{(q)}}{T_I^{(c)}} = \frac{\sqrt{\pi^3/2}}{\eta_1 \eta_2 P(q) T_0^{(c)}} \frac{a_L^2 \rho_L^2}{A_T' A_1} |T(\rho_1)|^2
\]

- Depending on parameter values, comparison may favor either source.

- **BUT**, broadband biphoton source is extremely vulnerable to background light, whereas narrowband pseudothermal source is not.

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Spatial Light Modulator Ghost Imaging

Spatial Light Modulator Ghost Imaging

- Assume SLM is $(2M+1) \times (2M+1)$ array
  - $d \times d$ pixels, 100% fill factor, $D = (2M+1)d$, $M \gg 1$
- Apply random phase modulation to each pixel
- Measurement plane spatial correlation function

$$K'(\rho_1, \rho_2) = \frac{P}{2} \left( \frac{d^2}{D \lambda_0 L} \right)^2 e^{ik_0(\rho_2^2 - \rho_1^2)/2L}$$

$$\times \left( \prod_{u=x,y} \frac{\sin(k_0 du_1/2L)}{k_0 du_1/2L} \frac{\sin(k_0 du_2/2L)}{k_0 du_2/2L} \right)$$

$$\times \left( \prod_{u=x,y} \frac{\sin[k_0 D(u_1 - u_2)/2L]}{\sin[k_0 d(u_1 - u_2)/2L]} \right)$$
Spatial Light Modulator Ghost Imaging

- Photocurrent cross-correlation function

\[ C(\rho_1) = q^2 \eta_1 \eta_2 A_1 K'(\rho_1, \rho_1) \int_{A_2} d\rho K'(\rho, \rho) |T(\rho)|^2 \]

\[ + q^2 \eta_1 \eta_2 A_1 \int_{A_2} d\rho |K'(\rho_1, \rho)|^2 |T(\rho)|^2 \]

- Assume object lies within \( \lambda_0 L/d \) field of view

- Ghost image has spatial resolution \( \lambda_0 L/D \)

- Featureless background can be eliminated
  - use DC block on either photodetector
Computational Ghost Imaging

- Spatial light modulator ghost imaging can use deterministic phase modulation:

- Evaluate diffraction integral off-line in advance

- Obtain single-beam ghost image

  \[
  \text{field of view} = \lambda_0 L/d, \text{ spatial resolution} = \lambda_0 L/D
  \]

Computational Ghost Imaging

- One light beam and one photodetector
  - no nonlocal two-photon interference can occur

- Depth of focus for range-spread reflectance
  - pseudo thermal case
    \[ |\Delta L|/L = 4L/k_0 a_0^2 \ll 1 \text{ in near field of cw laser} \]
    - each focal region must be imaged separately

  - computational case
    \[ |\Delta L|/L \approx L/k_0 D^2 \ll 1 \text{ in near field of cw laser} \]
    - many focal regions may be imaged at once
What about Atmospheric Turbulence?

- cw laser
- rotating ground glass
- $E(\rho, t)$
- $E_S(\rho, t)$
- $E_1(\rho, t)$
- $E_R(\rho, t)$
- $L$-m atmospheric propagation
- object, $T(\rho)$
- bucket detector (fixed)
- $i_2(t)$
- pinhole detector, center $\rho_1$ (scanning)
- $i_1(t)$
- correlator
- $C(\rho_1)$
Potential for Aberration Immunity

- Turbulence only between object and bucket
  - no loss of resolution

- Identical turbulence on both paths
  - no loss of resolution

- Statistically identical turbulence on both paths
  - resolution becomes turbulence limited

- Turbulence only between source and bucket
  - resolution becomes turbulence limited

- Turbulence only between source and pinhole detector
  - resolution becomes turbulence limited
**Discussion**

- Partially coherent light creates speckle patterns
  - speckle size ~ wavelength x path length/source size

- Ghost imaging is speckle pattern cross correlation
  - high-resolution images require *very small* speckles
Discussion

- Active two-beam ghost imaging
  - uses active illuminator to cast correlated speckle patterns
  - biphoton source: low brightness, low flux
  - pseudothermal source: high brightness, high flux
  - SLM source: controllable spatial coherence

- Active single-beam ghost imaging
  - uses precomputed high-resolution speckle pattern
  - only needs a bucket detector
  - can ghost image at wavelengths for which cameras unavailable

- Passive ghost imaging
  - uses natural-illumination speckle patterns
  - broadband operation yields very low image contrast
  - passive imaging without beam splitter requires very large speckles

- Ghost imaging has limited potential for aberration immunity
Other Work...

- **Far-field diffraction pattern imaging**

  $L$-m free space propagation (far field)

  - Type-II
    - Type-II SPDC
    - PBS
    - Transmission mask, $T(\rho)$
    - Far-field diffraction pattern imaging
    - Two-photon imaging

  - Pinhole detector, center $\rho$
  - Correlator $C(\rho)$

- **Two-photon imaging**

  - Type-II
    - Type-II SPDC
    - PBS
    - Transmission mask, $T(\rho)$

Erkmen & Shapiro, *PRA* (2008)
Future Work...

- Franco Wong will collaborate on experiments
- Two 512 x 512 SLMs have been purchased
- Quantitative ghost imaging experiments will be performed
- Will study field-of-view, resolution, and signal-to-noise ratio
- Will study classical phase-insensitive noise
- Will study classical phase-sensitive noise
- Will study nonclassical phase-sensitive noise