Experiments with Diffraction

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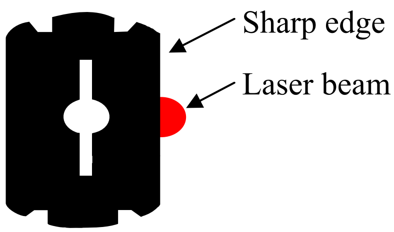
What is diffraction?
When parallel waves of light are obstructed by a very small object (i.e. sharp edge, slit, wire, etc.), the waves spread around the edges of the obstruction and interfere, resulting in a pattern of dark and light fringes.

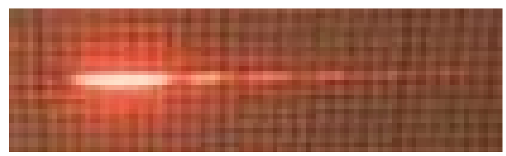
What does diffraction look like?
When light diffracts off of the edge of an object, it creates a pattern of light referred to as a *diffraction pattern*.

If a monochromatic light source, such as a laser, is used to observe diffraction, below are some examples of *diffraction patterns* that are created by certain objects:

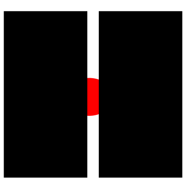
OBJECT | **DIFFRACTION PATTERN**

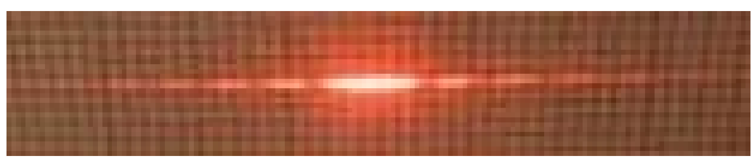
- **Sharp edge (i.e. razor blade)**



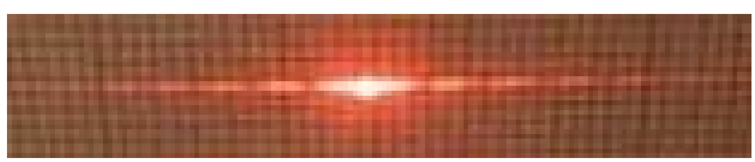


- **Slit**

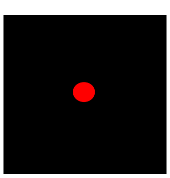




- **Wire**



- **Circular hole**

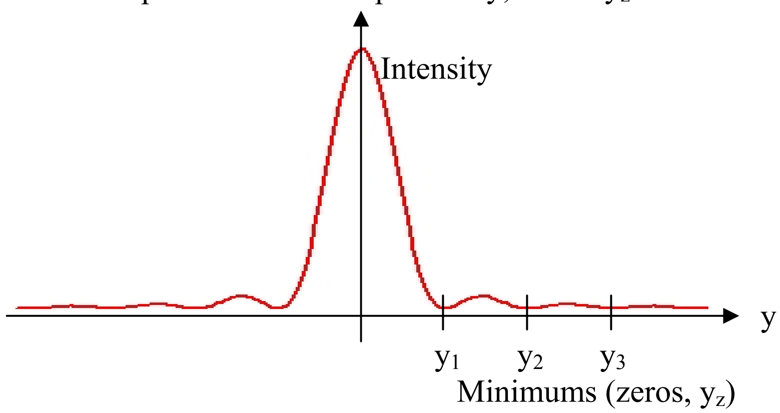


How can I determine the thickness of an object if I know certain dimensions of my diffraction pattern?

The intensity distribution for a diffraction pattern from a single slit is described mathematically as a *sinc* function where:

$$Intensity = \left(\frac{\sin(y)}{y}\right)^2$$

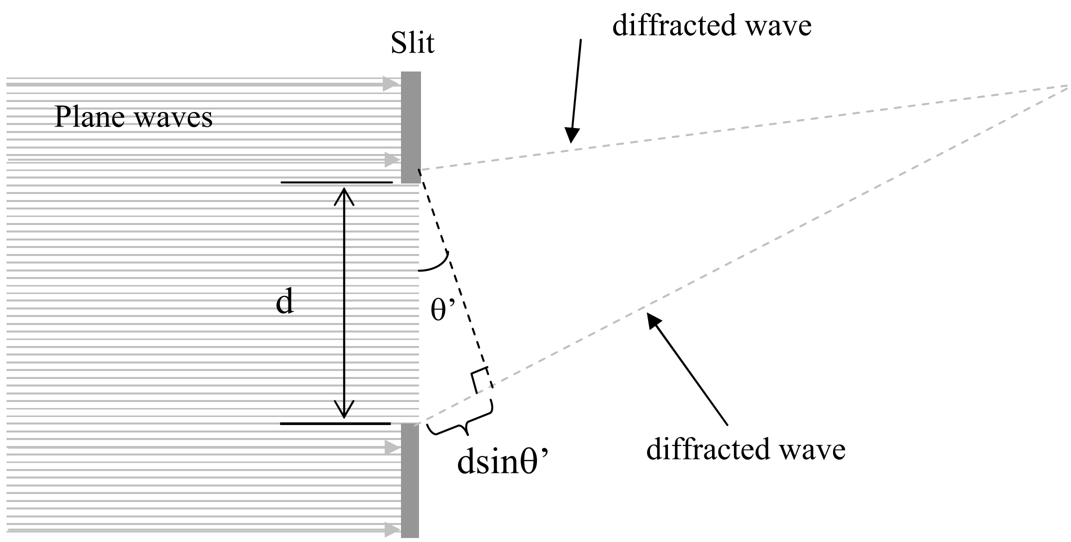
The intensity looks like the plot below versus position y, where y_z are the minimums (or zeros):



Minimums are caused by the destructive interference of plane waves diffracting off the edges of the slit. Destructive interference happens when two plane waves are out of phase to one another. When the phase difference, β, of two plane waves are equal to multiples of π, then a minimum occurs. We have the following:

$$\beta = \frac{\pi d}{\lambda}\sin\theta', \qquad \text{and zeros occur when } \beta = z\pi \text{ when } z = \pm1, \pm2, \pm3,\ldots$$

thus, zeros will occur when, $z\lambda = d\sin\theta'$

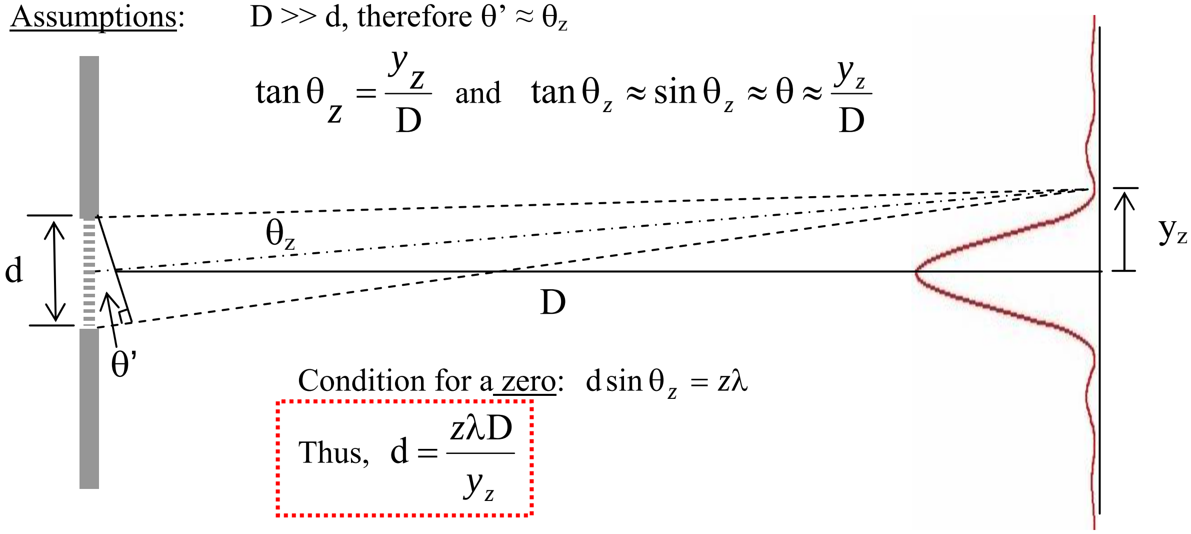


Geometrically, we can derive the relationship between the diameter of the slit, d, and the distance to a minimum or zero in the sinc function, y_z. Here is how we set up the problem, we have:

- d = slit diameter
- z = a minimum or zero (±1, ±2, ±3,…)
- θ' = diffracted wave angle
- λ = laser wavelength
- θ_z = sinc(y) function angle for zero
- D = distance from slit to screen
- y_z = distance from center of diffraction pattern to a minimum or zero.

<u>Assumptions</u>: D >> d, therefore $\theta' \approx \theta_z$

$$\tan\theta_z = \frac{y_z}{D} \quad \text{and} \quad \tan\theta_z \approx \sin\theta_z \approx \theta \approx \frac{y_z}{D}$$



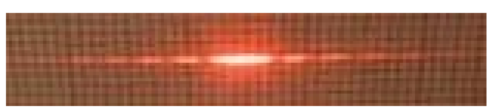
Condition for a <u>zero</u>: $d\sin\theta_z = z\lambda$

$$\text{Thus,} \quad d = \frac{z\lambda D}{y_z}$$

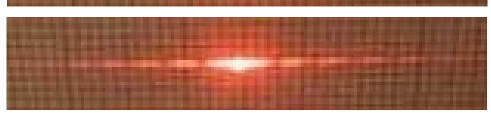
Can one use the same formula for measuring the thickness of a wire or human hair?

Yes. The distance from the minimums to the center of the diffraction pattern is still the same for the diffraction pattern caused by a wire of the same thickness as a slit. The only difference is that the center of the diffraction pattern looks brighter because the percentage of the laser beam that is not diffracted by the wire add to the intensity of the center of the pattern.

Diffraction pattern from a slit →



Diffraction pattern from a wire →



****One can also calculate the wavelength, λ, of their laser if they measure y_z from a diffraction pattern of a slit or wire of known d.**

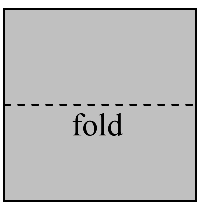
How can you demonstrate the relationship between the diffraction pattern from a slit or a wire and the thickness of the slit or wire?

If we solved the previous equation for y_z, (assuming that we will look at the first minimum in the pattern, z=1) we get:

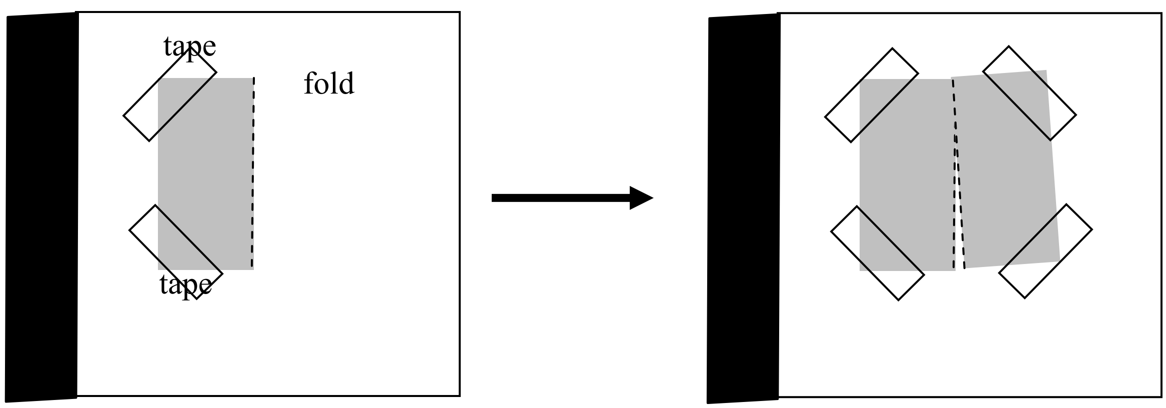
$$y = \frac{\lambda D}{d}$$

You can make a variable slit by using the following:

- (2) 1" square pieces of aluminum foil
- scotch tape
- 1 empty CD jewel case.



1. Fold each piece of aluminum foil in half and flatten the fold with something non-metallic (such as plastic scissor handles). Flattening the fold should create a sharper edge.
2. Tape one of the folded and flattened pieces of aluminum foil to the inside of the CD case. (see below left)



3. Overlap the other piece of folded aluminum foil (such that the folds are facing one another), and tilt it slightly until a variable thickness slit is formed. The slit should be very thin at the top and wider at the bottom. (see above right)
4. Stand the CD case upright, and using a laser, shine it through the slit so that the diffracted pattern can be seen on a wall, black board or screen behind the CD Case. Watch how the pattern changes in width as you move the laser up and down along the axis of the slit.

As the slit gets bigger, what happens to the width of the diffraction pattern? (Remember from the equation at the top of the page, that the thickness of the slit, d, is in the demonimator!)

Next ask, what would happen to the width of the pattern if the thickness of the slit stayed the same, but the wavelength of the laser, λ, would change?

What would be a common object to compare the thicknesses using diffraction?

A human hair is ideal to compare in a classroom!

For middle schoolers: students can compare the width of the diffraction pattern for different hairs measured. Keep the laser and the distance, D, to the wall or screen the same. Tape various students hairs vertically to the inside of a CD case (make sure you remember which hair is which!) Move the CD case so that each hair is illuminated in turn and have the students measure the width of the diffraction pattern from the center to the first minimum. Who has the thickest hair in the class based on the measurements? Who has the thinnest?

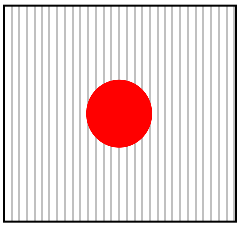
For high schoolers: students can measure the diffraction pattern with hairs set up the same as the in middle schoolers experiment, but, they can enter the measured data to actually calculate the thickness of each hair measured. Students can then determine the average hair thickness in their classroom and the standard deviation. Does curly hair tend to be thicker than straight hair? What about blonde hair and brown hair – any difference?**

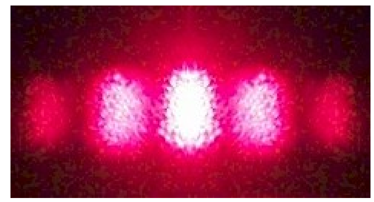
** for actual hair thickness measurements, the students will need to determine the wavelength of their laser. This can be accomplished by measuring the width of the diffraction pattern by using a wire of known thickness OR using a Compact Disc in the final experiment listed in this lesson plan.

What does diffraction look like for an object with a periodic structure?
If a *laser* is used to observe diffraction, below are some examples of *diffraction patterns* that are created by certain objects with repeating patterns:

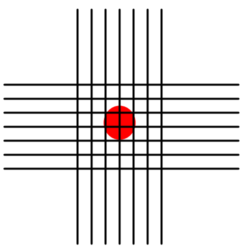
OBJECT (looking end-on) **DIFFRACTION PATTERN**

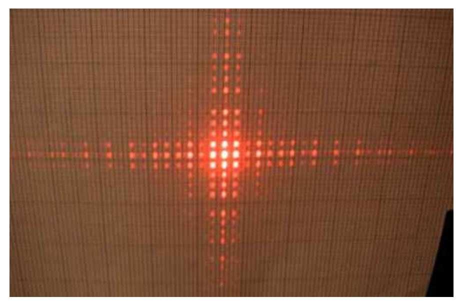
- **Grating**





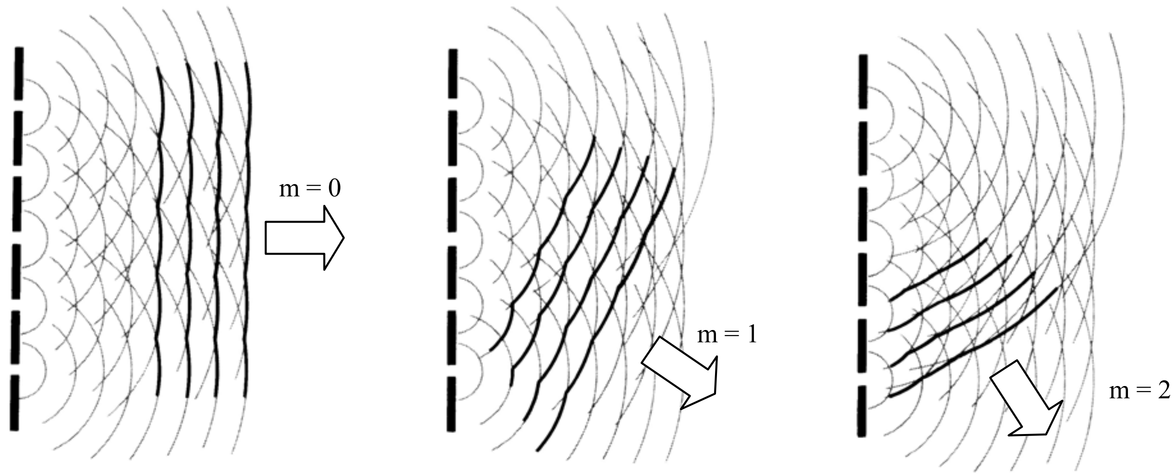
- **Mesh**





What is the relationship between the diffraction pattern and something with a periodic or repetitive structure like a grating?

The diffraction pattern from a grating differs from the pattern from and individual object. Let's get a sense of the wave behavior from a number of slits combined. If we look at the combined wavelets shown by the figure below, one can see different *orders*, m, of wavelets moving away from the slits.

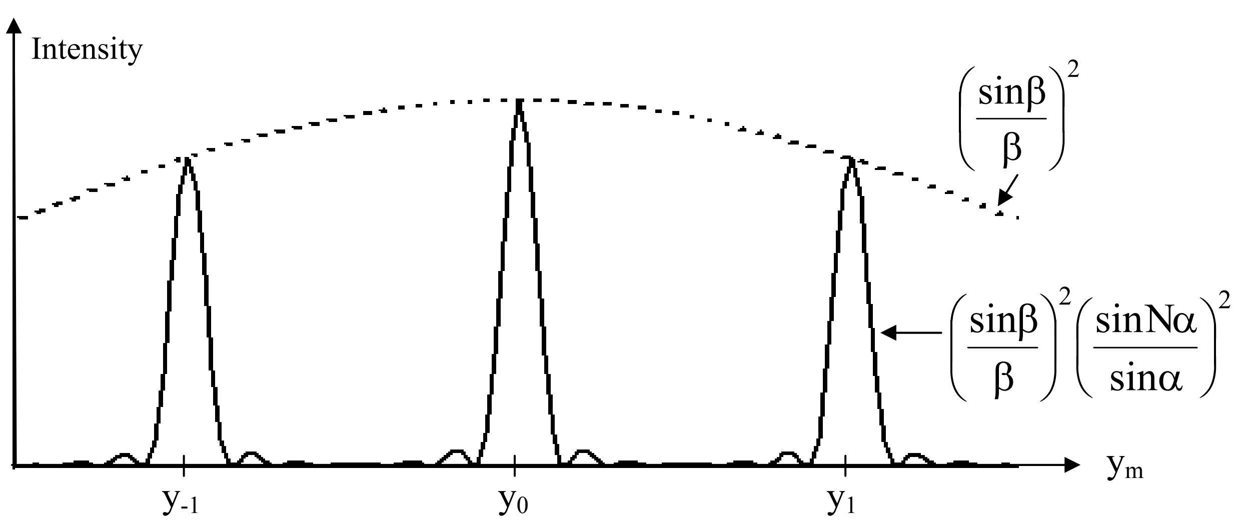


It has been shown previously that the shape of a diffraction pattern intensity from a single slit. When plane waves diffracted from multiple slits (# = N), of equal distance apart, are combined, the diffraction pattern get more complicated mathematically. The equation for the diffraction pattern intensity becomes:

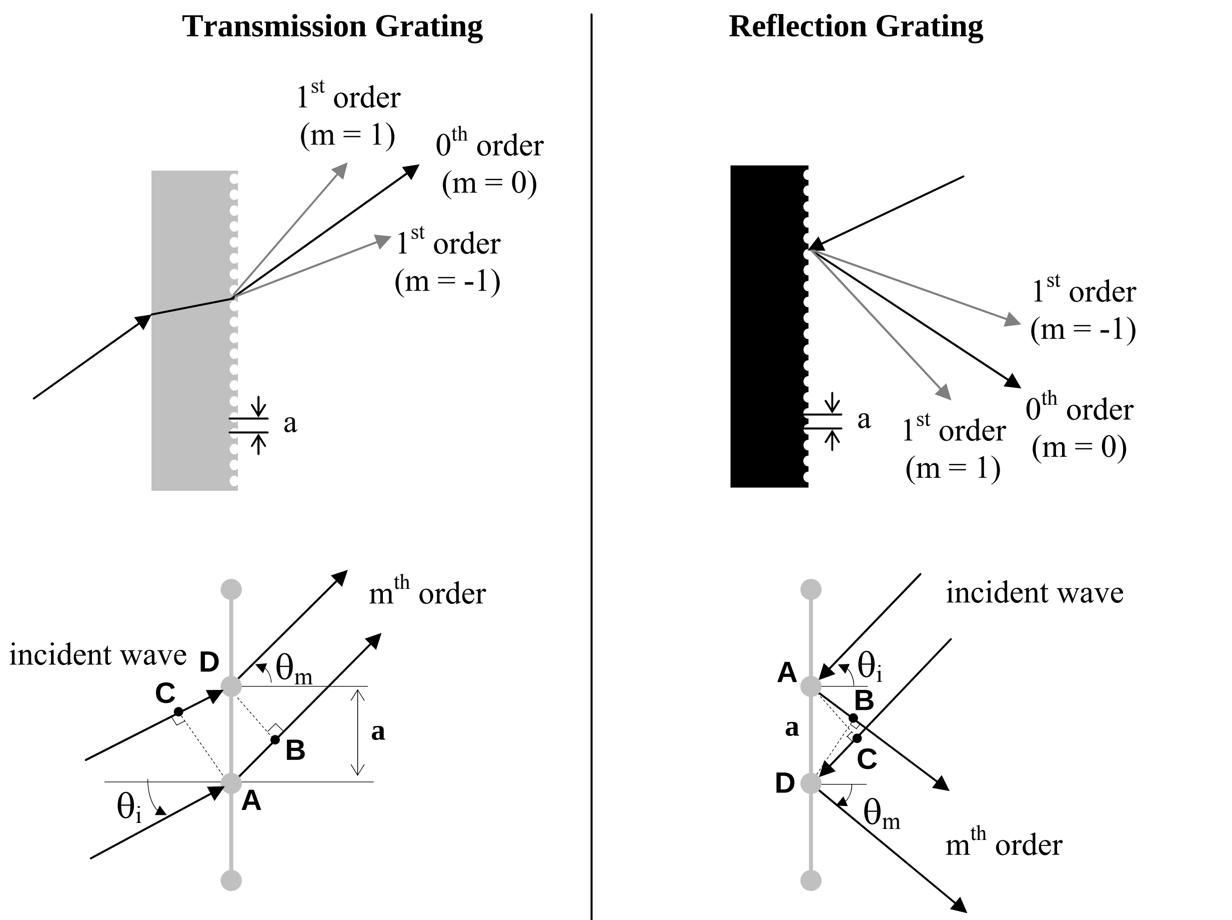
$$\text{Intensity} \rightarrow \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

where β = phase difference between diffracted waves from individual slit, and
α = phase difference between waves diffracted off of N slits

A generalized plot of the intensity is shown below.



Let's look at the geometry of two types of gratings, one that transmits and one that reflects.



$$\mathrm{AB} - \mathrm{CD} = a(\sin\theta_m - \sin\theta_i) \qquad\qquad \mathrm{AB} - \mathrm{CD} = a(\sin\theta_m - \sin\theta_i)$$

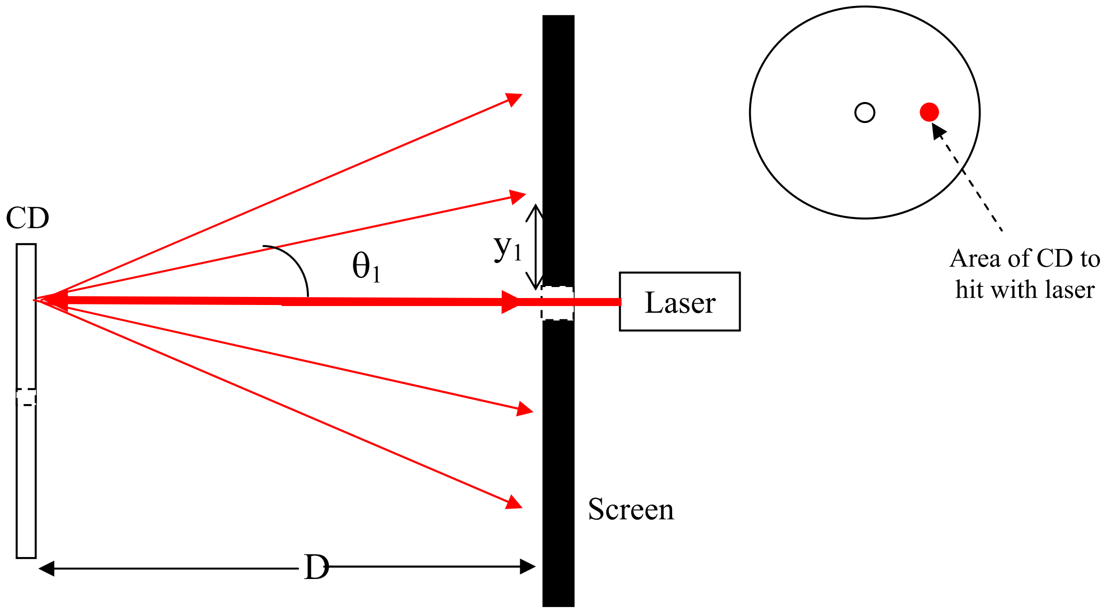
When taking measurements on the diffraction pattern from a grating, one would measure the distances between the central order (or 0th order) and higher order maxima. Therefore, the measured areas of the pattern are where there is constructive interference. Maxima created by constructive interference occur when the difference in phase, α, is a multiple of π. At normal incidence, when $\theta_i = 0$, we have the following:

$$\alpha = \frac{\pi a}{\lambda}\sin\theta_m, \text{ and maxima occur when } \alpha = m\pi \text{ when } m = 0, \pm1, \pm2, \ldots$$

thus, maxima will occur when, $m\lambda = a\sin\theta_m$ ← **The Grating Equation**

Can one use a Compact Disc like a reflection grating?

Absolutely!! One can use a compact disc to determine the wavelength of your laser pointer. First, set your laser pointer up behind a cardboard screen with a hole in it for the laser to pass through. Then place the CD in the path of the laser (in the location indicated) so that the 0^{th} order of the diffracted laser beam reflects back through the hole in the screen. (note: you can use the CD case to hold the CD upright. Notice the area of the CD that should be illuminated by the laser.)



We have,

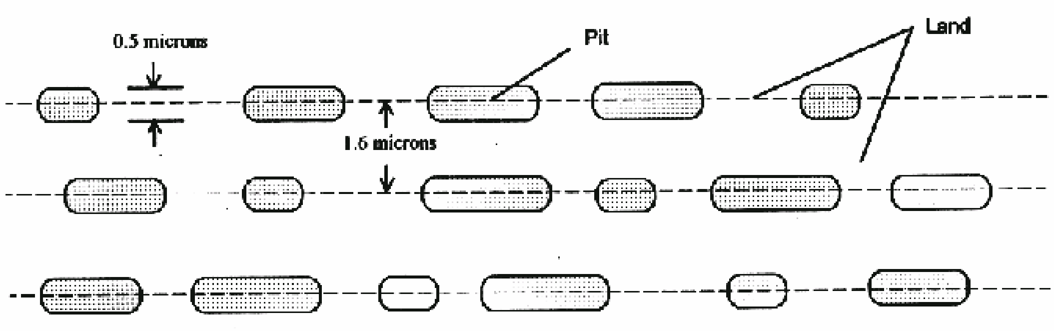
D = distance from CD to screen.

y_1 = distance from central beam to 1^{st} diffracted order.

θ_1 = angle of 1^{st} diffracted order.

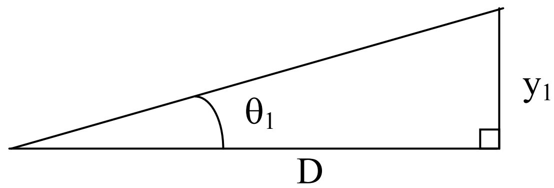
λ = laser wavelength

a = CD pit spacing = 1.6×10^{-6} meters (shown below)



Geometry of Compact Disc optical data (pit spacing)

Now, if we look at the geometry of our set-up, we have:



If your students are familiar with trigonometry, then:

$$\sin\theta_1 = \sin\left(\tan^{-1}\frac{y_1}{D}\right)$$

If your students are not familiar with trigonometry, then:

$$\sin\theta_1 = \frac{y_1}{\sqrt{D^2 + y_1^2}}$$

and, $\lambda = a\sin\theta_1$

Reference

1. Hecht, Eugene, **Optics**, 2nd Ed, Addison Wesley, 1987.