SELF-INDUCED TRANSPARENCY IN SELF-CHIRPED MEDIA

C.R. STROUD, Jr.
The Institute of Optics, University of Rochester, Rochester, NY 14627, USA

C.M. BOWDEN
Research Directorate, AMSMI-RD-RE-OP, Research, Development, and Engineering Center, U.S. Army Missile Command, Redstone Arsenal, AL 35898-5248, USA

and

L. ALLEN
North East London Polytechnic, Dagenham, Essex RM8 2AS, UK

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Resonant absorbing media having many atoms within a cubic resonance wavelength can exhibit large frequency chirps in pulse propagation due to dipole–dipole interactions among the atoms. Analytic solutions are obtained for invariant pulse propagation in such a medium consisting of interacting two-level atoms. Departures from the McCall–Hahn results include nonhyperbolic secant pulse shapes and different than 2π-area pulses. In addition, the excitation of the medium is comparatively suppressed for the same group velocity. The results are interpreted to suggest new experiments.

Recent calculations have shown that electromagnetic field propagation in a medium consisting of near resonant, two-level atoms can cause significant dynamic renormalization of the single atom resonance frequency [1–4]. This renormalization appears as an inversion-dependent chirp of the resonance frequency in the Bloch equations describing the two-level atoms [2], and is caused by coherent interaction among the nearby dipoles. It is shown that for sufficiently large density and oscillator strength, intrinsic optical bistability (IOB) occurs [1–4], (i.e., optical bistability which does not depend upon optical feedback, as in a high-Q cavity) even for propagation lengths much smaller than an atomic resonance wavelength [2–4].

Here, we consider the conditions for invariant pulse propagation in such media by analogy with the well known phenomenon of self-induced transparency (SIT) deduced from the usual Maxwell-Bloch formulation [5] and thoroughly investigated experimentally [6].

Our set of modified Maxwell-Bloch equations [1] take the form

$$\partial u/\partial\tau=[\Delta-\epsilon w]v\,, \tag{1a}$$

$$\partial v/\partial\tau=\Omega w-[\Delta-\epsilon w]u\,, \tag{1b}$$

$$\partial w/\partial\tau=-\Omega v\,, \tag{1c}$$

$$\partial\Omega/\partial\tau=-\beta^2 v\,, \tag{1d}$$

where

$$u^2+v^2+w^2=1\,. \tag{2}$$

Here, u and v are the slowly varying in-phase and out-of-phase components of the dipole moment and w is the atomic excitation, $-1\leqslant w\leqslant +1$, where $w=-1$ is the atomic ground state and $w=+1$ corresponds to complete inversion. Also, Δ is the detuning of the field frequency ω from the single atom resonance frequency ω_a, $\Delta=\omega-\omega_a$. Eq. (1d) is the equation for the slowly varying electric field amplitude $E=\hbar\Omega/d$, where d is the matrix element of the transition dipole moment. The time variable τ, is retarded time such that $\tau = t - 2/V$ where V is the velocity of the pulse. Finally,

$$\beta^2 = \frac{2\pi d\omega p}{\hbar cn[1/V - 1/U]}, \tag{3}$$

where U is the phase velocity of the field in the medium, n is the real part of the index of refraction, ρ is the density of atoms, c is the velocity of light in vacuum.

Eqs. (1) are the McCall–Hahn equations [5] in the limit $\epsilon \to 0$. The atomic frequency renormalization parameter, ϵ, was derived previously [2–4], and is given by [1]

$$\epsilon = \tfrac{7}{4}\pi\rho y_0 c^3/\omega_a^3, \tag{4}$$

where y_0 is the spontaneous decay rate for a single atom. The value for ϵ, eq. (4), can easily be ten to twenty times the natural atomic line width; the threshold value of the ratio ϵ/y_0, for IOB is six [2–4]. The magnitude of ϵ is determined by the strength of the interatomic dipole–dipole interaction in the material [2–4]. Eqs. (1) result from a fully quantum mechanical approach [2] as well as from a corresponding semiclassical treatment [3,4]. The effect of the terms having coefficient ϵ in eqs. (1) is to cause dynamic (excitation, w, dependent) chirp in the pulse propagation. This produces the possibility that invariant pulse propagation may occur with very low-to-moderate atomic excitation, even when the field frequency is tuned to resonance in the medium.

For the ususal boundary conditions for self-induced transparency, eqs. (1) have the solution

$$\Omega(\tau) = \frac{\Omega_0 \cos(\frac{1}{2}\phi)}{[\cosh^2\lambda\tau - \sin^2(\frac{1}{2}\phi)]^{1/2}}, \tag{5}$$

with the set of parameters

$$\Omega_0 = \lambda/g, \tag{6a}$$

$$b = \tfrac{1}{4}[1 - (4\epsilon/3\beta^2)(\Delta + \epsilon)], \tag{6b}$$

$$f = \epsilon\lambda/3\beta^2, \tag{6c}$$

$$g = (b^2 + f^2)^{1/4}, \tag{6d}$$

$$\lambda = [\beta^2 - (\Delta + \epsilon)^2]^{1/2}, \tag{6e}$$

and

$$\tan\phi = f/b. \tag{7}$$

Physical solutions require λ to be real. From eq. (6e), this is satisfied for

$$-(\beta + \epsilon) < \Delta < (\beta - \epsilon). \tag{8}$$

The pulse amplitude, eq. (5), has very nearly a hyperbolic secant shape. Because of the modulation of the resonance frequency caused by the terms in ϵ in eqs. (1), the pulse shape differs from that of a hyperbolic secant by the second term in the square root in the denominator of eq. (5). The Bloch vector follows a complicated trajectory on the unit sphere, reaching a maximum excitation given by

$$w_{max} = -1 + \Omega_0^2/2\beta^2. \tag{9}$$

In the limit, $\epsilon \ll \beta$, the solution, (5), is exactly the pulse amplitude of McCall and Hahn [5],

$$\Omega(\tau) = 2\lambda \operatorname{sech} \lambda\tau, \quad \epsilon \ll \beta, \quad w_{max} = +1. \tag{10}$$

If we choose ϵ to be ten homogeneous line widths and the pulse length to be one-tenth the spontaneous emission lifetime, and tune to resonance for the medium, i.e., set $\Delta = -\epsilon$, the maximum excitation during the pulse is $w_{max} = 1/5$. If, on the other hand, we choose ϵ to be one hundred times the line width with the same pulse length, then $w_{max} = -0.975$, in which case the atomic medium is barely excited. There is SIT in the sense of pulse delay and in the sense of a lack of loss of pulse energy as it propagates through the medium, but for the case of initial resonance with the medium, the atoms are never fully inverted.

Another unique feature of the pulses of eq. (5) is the area, which is given by

$$A = (2\Omega_0/\lambda)\cos(\tfrac{1}{2}\phi)\, K(k^2), \tag{11}$$

where $K(k^2)$ is a complete elliptic integral of the first kind and k is the modulus,

$$k = \sin(\tfrac{1}{2}\phi). \tag{12}$$

The area, A, is 2π only for the condition, $\epsilon = \Delta = 0$. In general, for tuning to intial resonance for the medium, $\Delta = -\epsilon$, it is less than 2π. For the values of the parameters of the two examples discussed above which gave $w_{max} = 1/5$, $w_{max} = -0.975$, corresponding to $\epsilon/\beta = 1$ and 10, respectively; the area from eq. (11) for each case is $A \approx 3\pi/2$, and $A \approx \pi/2$. The analogy here with the McCall–Hahn case of resonant pulse propagation is that the pulse be tuned to initial resonance with the medium, i.e., $\Delta = -\epsilon$. The contrast with the McCall–Hanh result of the pulse area is governed, under these conditions, entirely by the ratio ϵ/β. To date it would appear that pulse area measurements have not been carried out with sufficient accuracy to distinguish between π and 2π pulses, and thus there is insufficient data to preclude the possibility of such pulses [6].

A unique particular solution to eqs. (1) is obtained when $\Delta=\beta-\epsilon$, and $\epsilon/\beta>3/4$. Thus, from eq. (6e), $\lambda=0$. For this case we find that the intensity has a lorentzian envelope,

$$\Omega^2(\tau)=(6\beta^2/\epsilon)\,T\,[1/(\tau^2+T^2)]\,, \tag{13}$$

where

$$T=2\epsilon/[\beta(4\epsilon-3\beta)]\,. \tag{14}$$

The energy in such a pulse is finite, but the area is infinite as in the case of the elliptic function solutions of the conventional equations [6–8]. The lorentizian envelope has also been obtained for soliton solutions in a model for stimulated Raman scattering under specialized conditions [9,10].

From these results, it is noted that when there is chirping of the resonance frequency due to interatomic interaction, it is possible to have self-induced transparency without achieving nearly full inversion of the atomic system, even for the field frequency at initial resonance with the medium. It would thus seem quite interesting to have new experiments carefully measuring the area of self-induced transparency pulses as a function of pulse duration.

It is noted that SIT pulses of the form identical to eq. (5) were obtained by Kujawski and Eberly [11] and by Kujawski [12] using a different model. Their results stem from a perturbation expansion in the wave equation for the field, leading to a correction beyond the ususal slowly varying envelope and phase approximation, which causes a chirp in the pulse propagation. Due to the perturbation analysis their results are confined to very narrow pulses.

Beyond the direct application of our results in stimulating new experimental work in SIT, such experiments, together with the predictions presented here, can be used to determine the strength of the coherent dipole–dipole interaction and the magnitude, ϵ, of the corresponding frequency renormalization. This could serve as an independent verification of the predictions made earlier [1–4] in connection with IOB, under steady-state conditions, which is caused by the dipole–dipole interaction giving rise to ϵ.

Although there have been many reported experimental studies of IOB in many different materials corresponding to various mechanisms [13], with the exception of the work of Bohnert et al. [14], and Rossmann et al. [15], none have been interpretable according to an analytical model taken from first principles. The authors of refs. [14 and [15] observed IOB in single crystal platelets of CdS by tuning the incident field just below the band edge. Their results were interpreted by Schmidt et al. [16] as due to renormalization of the bandgap to lower frequencies caused by production of carriers in the material. The renormalization of the bandgap in GdS is due to Coulomb screening in the many-body interaction and is qualitatively quite similar to chirp in the resonance frequency discussed here.

Perhaps conditions consistent with our model presented here might be realized in laser field propagation in a system consisting of Rydberg atoms. Large oscillator strengths are characterisitic of Rydberg atoms and it is entirely possible to achieve ten to one hundred atoms per cubic wavelength. In such cases, ϵ can be made quite large. This would offer a comparison with suggested experiments in IOB as well [2], using the same medium.

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