

PREDICTIONS OF VIOLATIONS OF BELL'S INEQUALITY IN AN 8-PORT HOMODYNE DETECTOR

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A violation of Bell's inequality is predicted in the system of correlated electromagnetic field modes present in an optical 8-port homodyne detector. In this system all of the field modes are of the same polarization, and the correlated field modes are analyzed with phase sensitive homodyne detectors.

Since Bell's work [1] was published a number of experiments [2-11] have been undertaken to test Bell's inequality. Some of these experiments made use of correlated spin systems [10,11] such as the ones described by Bohm [12] and Bell [1]. However, the system used in the majority of these experiments [2-9] consisted of a pair of photons correlated via their polarizations. The correlated two-photon state is essentially the same as the spin singlet state. Such a state can be produced in the two photons emitted successively in a cascade atomic decay. The devices used to analyze the pair of photons are polarizing filters or polarizing beam splitters followed by photodetectors. These devices are precisely analogous to the Stern-Gerlach magnets used in Bohm's and Bell's gedanken experiments in that they facilitate the measurement of an arbitrary linear combination of the noncommuting observables associated with each photon polarization. Recently Ou, Hong and Mandel [13] have discovered a new way to produce the photon-polarization singlet state by using the output of a parametric down converter. When this output is prepared using a polarizing beam splitter the desired singlet state is produced. Once the correlated state is created the experiment proceeds the same as the other previous ones that used the cascade atomic decay. However, in another recent paper Ou and Mandel [14] have also shown that one can measure the same arbitrary linear combination of noncommuting observables associated with

a photon polarization by translating the position of a photodetector behind a polarizer oriented at 45° , rather than leaving the detector fixed and rotating the polarizer.

In this paper we consider a system of correlated electromagnetic field modes of the same polarization. Specifically the pair of modes emerging from an ordinary beam splitter. The degree of correlation between these modes is strongly dependent on the state of the field incident on the beam splitter. The correlated field modes are then analyzed with independently adjustable homodyne detectors. The homodyne detectors facilitate the measurement of the arbitrary linear combination of noncommuting observables associated with each of the correlated field modes. These devices replace the polarizing filters used in other previously considered systems. A theoretical analysis of this system leads to the derivation of a Bell inequality that holds for all possible input states. Then, by choosing different states of the field for the input to the correlating beam splitter, one can determine which states violate or satisfy the inequality.

The Bell inequality provides a means of testing the predictions of quantum mechanics against the predictions of a local hidden variable theory. We will first calculate the quantum predictions for the 8-port homodyne detector system. To do so consider the 8-port homodyne detector illustrated in fig. 1. The beam splitter labeled A is used to produce a corre-

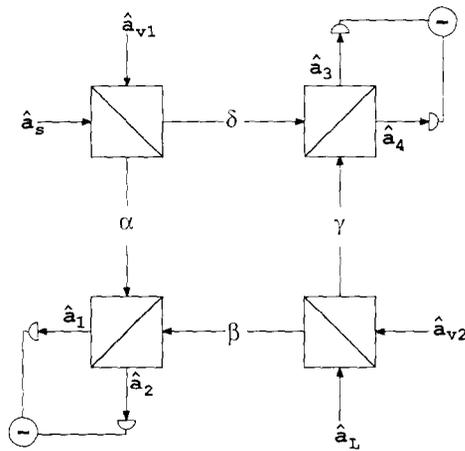


Fig. 1. Schematic diagram of an 8-port homodyne detector. Beam splitters B and D act as homodyning mixers. Beam splitter A is used to produce a correlated state in the two modes that connect beam splitters A to D and A to B. Beam splitter C is used to split the local oscillator into two modes, one for each homodyne detector. Annihilation operators \hat{a}_i and \hat{a}_L correspond to the signal and local oscillator modes respectively. The annihilation operators \hat{a}_{v1} and \hat{a}_{v2} correspond to vacuum modes. α , β , γ and δ are the optical path lengths between the beam splitters.

lated state. Beam splitters B and D act as the homodyne mixers and they are followed by detectors which measure the intensity of the four output modes with annihilation operators \hat{a}_1 through \hat{a}_4 . Beam splitter C acts to split the local oscillator mode with annihilation operator \hat{a}_L into two: one for each homodyne detector. The modes with annihilation operators \hat{a}_s , \hat{a}_{v1} , and \hat{a}_{v2} are the signal mode and vacuum modes respectively. The quantum mechanical prediction for the joint probability of detecting a photon at detectors i and j is given by the normally ordered expectation value

$$P_{ij} = K^2 \langle : \hat{n}_i \hat{n}_j : \rangle,$$

where K is a proportionality constant and $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$, so

$$P_{ij} = K^2 \langle : \hat{a}_i^\dagger \hat{a}_i \hat{a}_j^\dagger \hat{a}_j : \rangle. \quad (1)$$

Now consider the correlation function

$$E = P_{13} + P_{24} - P_{14} - P_{23}. \quad (2)$$

Using eq. (1) we can calculate the quantum mechanical prediction for this correlation function and obtain

$$E = K^2 \langle : (\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2) (\hat{a}_3^\dagger \hat{a}_3 - \hat{a}_4^\dagger \hat{a}_4) : \rangle. \quad (3)$$

Using the input/output relationship [15] for modes entering and exiting a beam splitter and keeping track of the phases accumulated during propagation between the beam splitters we can write the output mode annihilation operators in terms of the input mode annihilation operators. The result is

$$\hat{a}_1 = \frac{1}{2} [(\hat{a}_{v2} e^{i\beta} - \hat{a}_s e^{i\alpha}) + i(\hat{a}_L e^{i\beta} + \hat{a}_{v1} e^{i\alpha})], \quad (4a)$$

$$\hat{a}_2 = \frac{1}{2} [(\hat{a}_{v1} e^{i\alpha} - \hat{a}_L e^{i\beta}) + i(\hat{a}_s e^{i\alpha} + \hat{a}_{v2} e^{i\beta})], \quad (4b)$$

$$\hat{a}_3 = \frac{1}{2} [(\hat{a}_1 e^{i\gamma} - \hat{a}_{v1} e^{i\delta}) + i(\hat{a}_s e^{i\delta} + \hat{a}_{v2} e^{i\gamma})], \quad (4c)$$

$$\hat{a}_4 = \frac{1}{2} [(\hat{a}_s e^{i\delta} - \hat{a}_{v2} e^{i\gamma}) + i(\hat{a}_L e^{i\gamma} + \hat{a}_{v1} e^{i\delta})]. \quad (4d)$$

Substituting for $\hat{a}_1 - \hat{a}_4$ in eq. (3) with eqs. (4a)–(4d) and tracing over the vacuum states of the modes with annihilation operators \hat{a}_{v1} and \hat{a}_{v2} we have

$$E = E(\theta_1, \theta_2) = \frac{1}{4} K^2 \langle : [2\hat{a}_L^\dagger \hat{a}_L \hat{a}_s^\dagger \hat{a}_s \cos(\theta_1 - \theta_2) - \hat{a}_L^{\dagger 2} \hat{a}_s^2 e^{i(\theta_1 + \theta_2)} - \text{h.c.}] : \rangle, \quad (5)$$

where $\theta_1 = \alpha - \beta$ and $\theta_2 = \delta - \gamma$.

The predictions of the local hidden variable theory for this system are found by assuming that each of the joint probabilities, P_{ij} , that make up $E(\theta_1, \theta_2)$ is separable if one includes a dependence on some hidden variable λ that carries all information about correlations between the modes with annihilation operators \hat{a}_i and \hat{a}_j . This assumption along with the well known locality assumption [1] allows us to write

$$E(\theta_1, \theta_2) = \int_{\Lambda} d\lambda \rho(\lambda) [P_1(\theta_1, \lambda) - P_2(\theta_1, \lambda)] \times [P_3(\theta_2, \lambda) - P_4(\theta_2, \lambda)], \quad (6)$$

where the locality assumption has allowed us to eliminate the dependence of P_1 and P_2 on θ_2 and the dependence of P_3 and P_4 on θ_1 . In order to construct a Bell inequality for this correlation function we need to make what is known as the “no enhancement” assumption [16]. We will require that the following inequalities hold true:

$$0 \leq P_1(\theta_1, \lambda), P_2(\theta_1, \lambda) \leq P_1(-, \lambda) + P_2(-, \lambda) \quad (7)$$

and

$$0 \leq P_3(\theta_2, \lambda), P_4(\theta_2, \lambda) \leq P_3(-, \lambda) - P_4(-, \lambda), \quad (8)$$

where the dashes indicate independence of the probabilities on the phase angles θ_1 and θ_2 . This amounts to saying that the probability of there being a photon at one of the output ports of a lossless beam splitter is less than or equal to the sum of the probabilities of there being a photon at either of the output ports, and that the sum of the probabilities of there being a photon at either of the output ports is independent of the phases of the input modes. Now from eqs. (7) and (8) we can construct the inequalities

$$|P_1(\theta_1, \lambda) - P_2(\theta_1, \lambda)| \leq P_1(-, \lambda) + P_2(-, \lambda) \quad (9)$$

and

$$|P_3(\theta_2, \lambda) - P_4(\theta_2, \lambda)| \leq P_3(-, \lambda) + P_4(-, \lambda). \quad (10)$$

We can now apply an inequality derived by Clauser and Horne [16] that holds for a set of quantities such as those given in eqs. (9) and (10). This inequality is commonly used to derive a Bell inequality. Application of the Clauser and Horne inequality results in the following Bell inequality for the 8-port homodyne detector system:

$$|E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2')| \leq 2N, \quad (11)$$

where we have let $N = P_{13}(-, -) + P_{14}(-, -) + P_{23}(-, -) + P_{24}(-, -)$ and where $E(\theta_1, \theta_2)$ is given by eq. (5) or eq. (6). The primed angles in eq. (11) indicate alternative settings for the path lengths between the beam splitters in the 8-port homodyne detector. The quantum mechanical prediction for the probabilities in the bound of eq. (11) can be calculated using eq. (1) to be

$$N = K^2 \langle : (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) (\hat{a}_3^\dagger \hat{a}_3 + \hat{a}_4^\dagger \hat{a}_4) : \rangle. \quad (12)$$

Then using eqs. (4a)-(4d) and tracing over the states of the vacuum modes we have

$$N = \frac{1}{4} K^2 \langle : (\hat{n}_s^2 + \hat{n}_L^2 + 2\hat{n}_s \hat{n}_L) : \rangle. \quad (13)$$

Now that we have derived the Bell inequality for this system we can determine which input states satisfy or violate the inequality. Assuming that the local oscillator and the signal modes are both in coherent states, $|v_L\rangle$ and $|v_s\rangle$ respectively, and using these states to evaluate the expectation values in eqs. (6) and (13) we have

$$E(\theta_1, \theta_2) = K^2 |v_L v_s|^2 \sin \theta_1 \sin \theta_2 \quad (14)$$

and

$$N = \frac{1}{4} K^2 (|v_L|^4 + |v_s|^4 + 2|v_L v_s|^2). \quad (15)$$

Substituting for these expressions in eq. (11) we have

$$|\sin \theta_1 (\sin \theta_2 - \sin \theta_2') + \sin \theta_1' (\sin \theta_2 + \sin \theta_2')| \leq \frac{1}{2} \left(\frac{|v_L|^2}{|v_s|^2} + \frac{|v_s|^2}{|v_L|^2} + 2 \right). \quad (16)$$

This is the result that would be obtained from a purely classical treatment of the fields in the 8-port homodyne detector. The variable terms in the bound, i.e. the average number of photons in the local oscillator and signal modes, if taken to be equal, cancel out resulting in the minimum possible value for the bound. It is easy to show, using the method of Clauser and Horne [16], that this inequality is always satisfied. Thus the predictions of the classical theory are accounted for by a local hidden variable theory.

Next we will assume that one of the modes, for instance the signal mode, is prepared in a number state, $|n_s\rangle$, and the local oscillator mode is in a coherent state, $|v_L\rangle$. Then using these states to evaluate the expectation values in eqs. (6) and (13) we have

$$E(\theta_1, \theta_2) = \frac{1}{2} K^2 n_s |v_L|^2 \cos(\theta_1 - \theta_2) \quad (17)$$

and

$$N = \frac{1}{4} K^2 [|v_L|^4 + n_s(n_s - 1) + 2|v_L|^2 n_s], \quad (18)$$

Substituting these into eq. (11) we have

$$|S| \leq \frac{|v_L|^2}{n_s} + \frac{n_s - 1}{|v_L|^2} + 2, \quad (19)$$

where we have let

$$S = \cos(\theta_1 - \theta_2) - \cos(\theta_1 - \theta_2') + \cos(\theta_1' - \theta_2) + \cos(\theta_1' - \theta_2'). \quad (20)$$

The quantity in eq. (20) commonly appears in the study of systems that demonstrate violations of Bell's inequality. If we let $\theta_1 - \theta_2 = \theta_1' - \theta_2 = \theta_1 - \theta_2' = \frac{1}{3}(\theta_1 - \theta_2') = \varphi$ in eq. (20) then we obtain

$$S = 3 \cos \varphi - \cos 3\varphi. \quad (21)$$

The maximum value for S is obtained by letting $\varphi = \frac{1}{3}\pi$ in eq. (21). The result is $S = 2\sqrt{2}$. If we now let $n_s = 1$ in eq. (19) then we have

$$|S| \leq 2 + |v_L|^2. \quad (22)$$

This inequality can be violated if the average number of photons in the local oscillator mode satisfies the inequality

$$0 < |v_L|^2 < 2(\sqrt{2}-1). \quad (23)$$

This corresponds to a very weak local oscillator with less than one photon on average occupying the mode. The violation of the inequality in eq. (22) implies that the correlated state produced when a single photon is split by an ordinary beam splitter cannot be explained with a local hidden variable theory.

Finally, let us consider the case where the signal and local oscillator modes are prepared in number states $|n_s\rangle$ and $|n_L\rangle$ respectively. Using these states to evaluate the expectation values in eqs. (6) and (13) we obtain

$$E(\theta_1 - \theta_2) = \frac{1}{2} K^2 n_L n_s \cos(\theta_1 - \theta_2) \quad (24)$$

and

$$N = \frac{1}{4} K^2 [n_L(n_L - 1) + n_s(n_s - 1) + 2n_s n_L]. \quad (25)$$

Substituting these into eq. (11) we have

$$|S| \leq \frac{n_L - 1}{n_s} + \frac{n_s - 1}{n_L} + 2, \quad (26)$$

where S is given by eq. (20). If we let $n_L = n_s = 1$ in eq. (26) the resulting inequality is exactly the same as that obtained in all of the previously studied systems designed to test Bell's inequality. This combination of input states results in the strongest violation possible using the 8-port homodyne detector. It should be noted that in this case there is no longer a distinction between the local oscillator and signal modes. Both are prepared in single photon number states and the modes emerging from beam splitters A and C, as illustrated in fig. 1, are correlated in precisely the same way.

We have shown that the 8-port homodyne detector can demonstrate the same kind of non-local correlation effects as those seen in many other previously studied systems designed to test Bell's inequality. However, the correlations present in the 8-port homodyne detector are between the outputs of independently adjustable homodyne detectors which measure the in-phase or in-quadrature amplitudes of the

correlated input fields. This is markedly different from all of the previously studied systems which made use of polarizing filters or beam splitters as analyzers [2-9,13].

The only major difficulty to overcome in order to perform an actual experiment to demonstrate a violation of Bell's inequality in an 8-port homodyne detector is the production of a single photon number state. Such a state can be produced in the spontaneous emission field of a single atom or in the output of a weakly pumped parametric down converter. Both of these methods have been used by Mandel and co-workers [17] in their study of non-classical states of the electromagnetic field.

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References

- [1] J.S. Bell, *Physics* 1 (1965) 195.
- [2] S.F. Freedman and J.F. Clauser, *Phys. Rev. Lett.* 28 (1972) 938.
- [3] R.A. Holt, Ph.D. dissertation, Harvard University, Cambridge, MA (1979).
- [4] J.F. Clauser, *Phys. Rev. Lett.* 36 (1976) 1223.
- [5] E. Fry and R. Thompson, *Phys. Rev. Lett.* 37 (1976) 465.
- [6] A. Aspect, P. Grangier and G. Roger, *Phys. Rev. Lett.* 47 (1981) 460.
- [7] A. Aspect, J. Dalibard and G. Roger, *Phys. Rev. Lett.* 49 (1982) 91.
- [8] A. Aspect, J. Dalibard and G. Roger, *Phys. Rev. Lett.* 49 (1982) 1804.
- [9] W. Pierrie, A.J. Duncan, H.J. Beyer and H. Kleinpoppen, *Phys. Rev. Lett.* 54 (1985) 1790.
- [10] L.R. Kasday, J.D. Ullman and C.S. Wu, *Nuovo Cimento B* 25 (1975) 633.
- [11] G. Faraci, D. Gutkowski, S. Notarrigo and A.R. Pennisi, *Nuovo Cimento Lett.* 9 (1974) 607.
- [12] D. Bohm, *Quantum theory* (Prentice-Hall, Englewood Cliffs, 1951) p. 614.
- [13] Z.Y. Ou, C.K. Hong and L. Mandel, *Phys. Lett. A* 122 (1987) 11.
- [14] Z.Y. Ou, C.K. Hong and L. Mandel, *Opt. Commun.* 67 (1988) 159.
- [15] Z.Y. Ou, C.K. Hong and L. Mandel, *Opt. Commun.* 63 (1987) 118.
- [16] J.F. Clauser and M.A. Horne, *Phys. Rev. D* 10 (1974) 526.
- [17] L. Mandel, *Phys. Scr. T* 12 (1986) 34.