Bell's inequalities for Rydberg atoms

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The analogy between two-level atoms and spins is used to show that Rydberg atoms provide a new medium through which Bell's inequalities may be studied. The low-efficiency particle detectors that have been used in experiments to test Bell's inequalities are replaced by nearly 100% efficient state-selective ionizers in Rydberg-atom experiments. Two methods for producing correlated states of pairs of atoms are discussed. Each method can be used to create correlated states of more than two atoms. A Bell inequality for a three-atom system is considered and compared with quantum-mechanical predictions. A violation of the inequality for the three-particle system is predicted.

In the past decade much progress has been made in turning the *gedanken* experiments of quantum theory into laboratory realizations. The paradoxes and puzzles have been subjected to rigorous tests. In particular, the Einstein–Podolsky–Rosen "paradox" has been carefully studied in the laboratory.[1–10] In their famous paper Einstein *et al.*[11] noted that a two-particle quantum system might be prepared in a correlated state such that, even when the two particles separate by large distances, a measurement performed on one of the particles affects the wave function of the entire system. This measurement then immediately changes the state of the second particle, even though there is no direct communication between the two particles. These correlated systems have been studied a great deal.[12–16] One result of these studies has been the development of a series of inequalities, Bell's inequalities.[17] These inequalities are relations that apparently all classical and local hidden-variable theories must satisfy, but which quantum theory need not satisfy. In a beautiful series of experiments[1–8] several groups have tested these inequalities, using the polarization of photons emitted successively in a cascade atomic decay for the measured property of a correlated two-particle system. These experiments have been impressive in their demonstration of violations of Bell's inequalities and in their agreement with the predictions of conventional quantum theory, but they have been handicapped by the lack of photodetectors with very high quantum efficiency.[6,16]

In this paper we show that there is an alternative to the use of particle spins and photon polarizations as systems to demonstrate violations of Bell's inequalities. The recent development of techniques for working with and detecting, with very high quantum efficiency, single atoms in Rydberg states[18–21] allows us to propose a system of Rydberg atoms as a new tool for the study of Bell's inequalities and the locality assumption.

The generalization of the Einstein–Podolsky–Rosen paradox and Bell's inequalities to the case of Rydberg atoms is most easily seen if we consider a statement of the problem in terms of a pair of spins. Bohm[12] formulated the problem in this form. A pair of spins is initially in a singlet state and then separates, with each passing through a separate Stern–Gerlach magnet, the first magnet oriented at angle θ_1 and the second at angle θ_2. Then each particle passes into a detector.

A two-level atom interacting with a resonant field is exactly analogous to a spin interacting with a magnetic field,[22] so the analysis in the case of spins can be carried over immediately to the atomic case. The spin is replaced by the two-level-atom Bloch vector,[23] and the angles θ_1 and θ_2 are replaced by the turning angles of the Bloch vectors produced when the atoms interact with resonant fields. Even the particle detectors carry over immediately to the state-selective ionization detectors commonly employed for Rydberg atoms.[24]

To see in detail how the analysis carries over, we consider an idealized Rydberg-atom experiment (see Fig. 1). Assume that two atoms, each traveling along a slightly different trajectory, enter a high-Q microwave cavity. Initially, each of the atoms is prepared in the lower of two Rydberg states that are resonantly coupled by the fundamental mode of the cavity. The cavity is cooled to eliminate blackbody photons, and the field initially contains one photon in the resonant mode. If the atoms each travel through the cavity together in a time corresponding to a $(2n + 1)\pi$-pulse excitation of the two-atom system, the two atoms will be left in the correlated state

$$|\Psi_c\rangle = \frac{1}{\sqrt{2}}(|+,-\rangle + |-,+\rangle). \tag{1}$$

In this expression, $|+,-\rangle$ is the state in which the first atom is in the upper Rydberg level and the second atom is in the lower Rydberg level, and $|-,+\rangle$ is the state in which the first atom is in the lower state and the second in the excited state. After leaving the cavity, the two atoms enter separate resonant microwave fields. The first atom has its Bloch vector rotated through an angle $\theta_1/2$ and the second through an independently specifiable angle $\theta_2/2$. The angle θ_i is equal to the product of the Rabi frequency and the interaction time for each field:

$$\theta_i = \Omega_i \tau_i. \tag{2}$$

By operating on the correlated state [Eq. (1)] with an operator that is the tensor product of the individual two-dimensional rotation operators for each field

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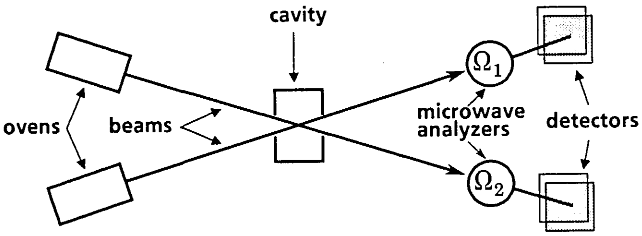

Fig. 1. Schematic representation of the ideal Rydberg-atom experiment. Ovens represent sources of Rydberg atoms prepared in the lower of two Rydberg states. The beams are atomic beams of Rydberg atoms. The open box (center) indicates the single-mode resonant cavity. The circles indicate interaction regions with resonant microwave fields having Rabi frequencies of Ω_1 and Ω_2, respectively. The overlapping squares indicate state-selective ionization detectors.

$$\hat{R} = \hat{R}_1 \otimes \hat{R}_2, \tag{3}$$

one finds the state of the pair of atoms after interacting with the fields. The new state is

$$|\Psi_\theta\rangle = \frac{i}{\sqrt{2}} (|\theta_1, \theta_2 - \pi\rangle + |\theta_1 - \pi, \theta_2\rangle), \tag{4}$$

where $|\theta_i\rangle$ is given by

$$|\theta_i\rangle = e^{-i\omega t/2} \cos(\theta_i/2)|+\rangle + ie^{i\omega t/2} \sin(\theta_i/2)|-\rangle. \tag{5}$$

Expanding the direct products of the theta states, one can rewrite Eq. (4) in terms of the direct-product eigenstates of the pair of atoms. Then we have

$$|\Psi_\theta\rangle = \frac{1}{\sqrt{2}} \left[i \sin\left(\frac{\theta_1 + \theta_2}{2}\right)(e^{-i\omega t}|+, +\rangle + e^{i\omega t}|-, -\rangle) + \cos\left(\frac{\theta_1 + \theta_2}{2}\right)(|+, -\rangle + |-, +\rangle)\right]. \tag{6}$$

This state corresponds to the state that one would find for the spin problem if the pair of spins were initially in the triplet state.

The inequality constructed by Bell[17] for the case of a nonidealized two-particle correlation is

$$-2 \le E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2') \le 2, \tag{7}$$

where E is the expectation value of the product of the individual atomic Hamiltonians and the primed angles are alternative settings of the analyzers. A quantum-mechanical violation of Bell's inequality can be demonstrated by calculating the expectation E with the state given in Eq. (6):

$$\langle H_1 H_2\rangle = \frac{\hbar^2\omega^2}{4} E(\theta_1, \theta_2) = -\frac{\hbar^2\omega^2}{4} \cos(\theta_1 + \theta_2). \tag{8}$$

Substituting Eq. (8) into inequality (7) and choosing appropriate values for θ_1, θ_1', θ_2, and θ_2', one can cause the inequality to yield a falsehood. For example, if $\theta_1 = \theta_2 = 0$, $\theta_1' = \pm(\pi/3)$, and $\theta_2' = \pi \pm (\pi/3)$, then expression (7) becomes $0.5 \le 0$. The question of what this false statement means with regard to locality, hidden variables, reality, and probability theory has been discussed at great length in the literature.[15,25–27]

The experimental tests of Bell's inequalities using low-energy photons have upheld quantum mechanics (with two possible exceptions[2,10]), yet all have been plagued with low detector efficiencies and lack of control over the trajectories of the photons being counted. Systems that incorporate correlated energy levels as a source of violations of Bell's inequalities must make use of energy-state-selective detectors. The state-selective ionizers commonly employed for the detection of atoms in Rydberg states have quantum efficiencies approaching 100%. Recently Drummond[28] addressed the problem of low detector efficiencies in photon experiments by considering a cooperative state consisting of N pairs of photons. This system demonstrates a violation of a Bell inequality and increases the signal to be detected in an experiment. The atomic system considered in this work also facilitates the study of cooperative states made up of more than two particles. For example, a three-atom-correlated state

$$|\Psi_c\rangle = \frac{1}{\sqrt{3}} (|+, -, -\rangle + |-, +, -\rangle + |-, -, +\rangle) \tag{9}$$

can be produced by sending three ground-state two-level atoms through a resonant cavity initially in a one-photon Fock state. Instead of using the greater number of particles to increase the detection efficiency, one can construct a new Bell inequality by independently analyzing all the correlated atoms. The triple of atoms is analyzed the same way as the pair previously described. Each atom has its Bloch vector rotated through an independently specifiable angle θ_i given by Eq. (2). The new quantum state is found by operating on the state in Eq. (9) with the tensor-product operator

$$\hat{R} = \hat{R}_1 \otimes \hat{R}_2 \otimes \hat{R}_3. \tag{10}$$

The state produced is

$$|\Psi_\theta\rangle = \frac{-1}{\sqrt{3}} [|\theta_1, \theta_2 - \pi, \theta_3 - \pi\rangle + |\theta_1 - \pi, \theta_2, \theta_3 - \pi\rangle + |\theta_1 - \pi, \theta_2 - \pi, \theta_3\rangle]. \tag{11}$$

The expectation value of the product of all three atomic Hamiltonians is

$$\langle H_1 H_2 H_3\rangle = \frac{\hbar^3\omega^3}{8} E(\theta_1, \theta_2, \theta_3), \tag{12}$$

and

$$E(\theta_1, \theta_2, \theta_3) = \tfrac{1}{3}(3 \cos\theta_1 \cos\theta_2 \cos\theta_3 - 2 \cos\theta_1 \sin\theta_2 \sin\theta_3 - 2 \sin\theta_1 \cos\theta_2 \sin\theta_3 - 2 \sin\theta_1 \sin\theta_2 \cos\theta_3). \tag{13}$$

Bell's method for constructing the inequality shown in expression (7), for the case of nonidealized correlations, can be generalized to apply to the case of a three-particle system. Using this technique, one obtains a family of inequalities. By choosing one of the inequalities and substituting into it the quantum-mechanical predictions for the three-particle system, one finds a violation of the inequality such as that found for the two-particle system. A suitable inequality is given by

$$-2 \le E(\theta_1, \theta_2, \theta_3) - E(\theta_1', \theta_2', \theta_3) + E(\theta_1', \theta_2', \theta_3') + E(\theta_1, \theta_2, \theta_3') \le 2. \tag{14}$$

By substituting Eq. (13) into expression (14) and letting $\theta_1 = \theta_2 = \theta_3 = 0$, one can find values for θ_1', θ_2', and θ_3' such that expression (14) yields a falsehood. For example, if $\theta_1' = \pi/6$, $\theta_2' = 7\pi/6$, and $\theta_3' = \pi/3$, then expression (14) becomes $7/24 \le 0$. The interpretation of the falsehood is changed only by the fact that the quantum correlation is now split among three particles rather than between two.

The realizability of an actual experiment to test Bell's inequalities using correlated two-level atoms and resonant fields as analyzers depends largely on the degree of correlation that can be produced between the atoms. The foremost difficulties with the method of producing correlated states suggested here is the requirement of a one-photon Fock state and the need for the atoms to be in the cavity at the same time. To our knowledge, no one has succeeded in producing a one-photon Fock state in the laboratory. However, a method of producing a few-photon Fock state has been demonstrated theoretically,[29] and the rapidly advancing experimental capabilities in this area suggest that the difficulties are surmountable. It is also true that other methods of producing these correlated states are possible. One such method would eliminate the difficulties stated above and would again use an arrangement much like the one depicted in Fig. 1. The field in the cavity is prepared in the vacuum state, and the atoms to be correlated are sent through one at a time with independent velocities. The first atom to go through the cavity is prepared in the upper of two Rydberg states that are resonantly coupled by the fundamental mode of the cavity, and the second atom is prepared in the lower state. The two-atom field state that is produced is given by

$$|\Psi\rangle = \cos\frac{\Phi_1}{2}|+,-,0\rangle - \sin\frac{\Phi_1}{2}\sin\frac{\Phi_2}{2}|-,+,0\rangle + \sin\frac{\Phi_1}{2}\cos\frac{\Phi_2}{2}|-,-,1\rangle, \tag{15}$$

where Φ_1 and Φ_2 are given by Eq. (2). By choosing $\Phi_1 = (n + ½)\pi$ and $\Phi_2 = (2n + 1)\pi$, Eq. (15) becomes

$$|\Psi\rangle = \pm\frac{1}{\sqrt{2}}(|+,-\rangle - |-,+\rangle). \tag{16}$$

The state in Eq. (16) is an antisymmetric correlated state that demonstrates a violation of Bell's inequalities in the same fashion as the correlated state of Eq. (1). If a third atom were coupled to the system described by the state in Eq. (15), then one could easily produce a three-atom correlated state by choosing the appropriate interaction times for all three atoms. The two-atom field state of Eq. (15) also demonstrates squeezing of the atomic SU(2) operators[30,31] for certain values of the interaction angles Φ_1 and Φ_2. The angles that produce maximum squeezing are not coincident with the angles that produce correlated states.

In conclusion, Rydberg atoms interacting with high-Q cavities provide an alternative system for studying Bell's inequalities and quantum measurement theory. Rydberg-atom systems offer methods of producing correlated states in a controlled or a stimulated fashion rather than by spontaneous processes. These new methods open up several avenues of investigation of Bell's inequalities and correlated states in both the theoretical and the experimental domains.

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