Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713191304

Collective entangled dark states

Hideomi Nihira [a]; C. R. Stroud Jr [a]
[a] The Institute of Optics, University of Rochester, Rochester, NY 14627, USA

Online Publication Date: 01 September 2007
To cite this Article: Nihira, Hideomi and Stroud Jr, C. R. (2007) 'Collective entangled dark states', Journal of Modern Optics, 54:13, 2121 - 2127
To link to this article: DOI: 10.1080/09500340701391449
URL: http://dx.doi.org/10.1080/09500340701391449

Journal of Modern Optics
Vol. 54, Nos. 13–15, 10 Sep–15 Oct 2007, 2121–2127

Collective entangled dark states

HIDEOMI NIHIRA* and C. R. STROUD JR
The Institute of Optics, University of Rochester,
Rochester, NY 14627, USA

(*Received 31 January 2007; in final form 19 March 2007*)

We describe a type of collective entangled dark state which exists when a group of many-level atoms interact simultaneously with both high-Q cavity modes and free space modes of the electromagnetic field. As an example we describe a collection of atoms with a level structure of the form of $J = 1/2 \to J = 1/2$ and show that there are states of this system in which the atoms remain trapped in a collective coherent superposition of ground state levels in spite of the presence of photons in resonant field modes. Efficient methods of generalization of these results to other atomic level structures and cavity fields are described.

1. Introduction

Peter L. Knight has made many contributions to the field of quantum optics. Amongst them is the study of atom–field entanglement and how it can be manipulated [1]. This paper presents the decoherence-free states of N two-level atoms inside a cavity and a procedure to manipulate those states. Our paper is an extension of this work in which we present decoherence-free states of collections of multi-level atoms. We direct our attention to those decoherence-free states which only contain ground states of the atoms thereby allowing us to eliminate the effects of spontaneous decay.

The regime of multiple atoms and a few photons in high-Q cavity modes is an interesting one which presents such varied phenomena as entanglement, superradiance, and coherent population trapping. We explore a system in which all of these effects occur simultaneously and produce an interesting decoherence-free state that may be of use in quantum information science.

In this paper we present the complete set of collective entangled dark states of a $J_{1/2} \to J_{1/2}$ system interacting with three cavity fields and a systematic method of obtaining these dark states for different types of atomic energy configurations. These dark states are immune to spontaneous decay and are entangled in both the atoms and the fields. The method introduced will effectively reduce the dimensionality of the space to make the calculations more manageable. The purpose here is to obtain the exact form of the set of collective entangled dark states by exploiting the symmetries of the problem.

*Corresponding author. Email: nihira@optics.rochester.edu

Journal of Modern Optics
ISSN 0950–0340 print/ISSN 1362–3044 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/09500340701391449

The system we consider is a cavity containing two $J_{1/2} \rightarrow J_{1/2}$ atomic systems interacting with three cavity modes. Furthermore, we allow only one excitation in this atom–cavity system. The dark states derived contain only atomic ground states which make the states immune to spontaneous decay. Also, the dark states are entangled in the atomic and field degrees of freedom. The single excitation in the cavity is distributed amongst the three cavity modes which are entangled with the atomic states.

Two closely related phenomena that initially sparked interest in collective quantum states are the superradiance and subradiance [2–4] in which radiation rates are modified from the single atom radiation rate due to the collection of atoms. The earliest paper considering collective atomic radiative effects is that of Dicke in his treatment of coherence in spontaneous emission [2]. In his paper he argues that the regular treatment of the spontaneous emission process in a gas as just a sum of individual atomic radiators is incorrect and that many of the results obtained are wrong. Although Dicke does not explicitly mention entanglement in his paper, the Dicke state,

$$|\Psi\rangle = \frac{1}{N^{1/2}} \sum_j |g_1, g_2, \ldots, e_j, \ldots, g_N\rangle, \tag{1}$$

where N is the number of atoms in the collection, g_i is the ith atom in the ground state, and e_j is the jth atom in the excited state, is indeed an entangled state. One property of the Dicke state is superradiance. This collective entangled state which has only one excitation and zero average dipole moment has a spontaneous emission probability proportional to N. The decay rate of the single atom in the collection is enhanced by a factor of N.

A more recent example of collective quantum systems is spin-squeezed states [5–7]. These states are of interest because of their sub-shot noise and entanglement characteristics [5]. Although the focus of this study is not to obtain dark states, they are important examples of the interest in radiative properties of collective quantum systems.

In this paper we take a look at collective entangled dark states of atoms and fields. Fleischhauer *et al.* [8] utilized the collective entangled dark state of N lambda systems as a quantum memory. In their paper one of the transitions of the lambda system is driven by a coherent field and the other transition interacts with a cavity mode field. The resulting collective dark state has a form similar to a Bell state. In our paper we take a look at a different system in which the atoms interact only with the cavity modes. Here we see that the resulting dark states do not take a form analogous to a Bell state. These states are entangled in both the atoms and the fields and yet are immune to spontaneous decay. Once prepared in these states the atoms and the fields of the system remain in these collective entangled dark states until collisions or cavity losses destroy the collective phase relations.

A more explicit use of entangled dark states in a high-Q cavity is given by [9]. In this paper the authors show that the entangled dark state of two atoms could serve as a fundamental building block of a quantum computer by demonstrating how an arbitrary conditional operation on the two qubits can be performed. The qubit states here are in a dark state and the coherence transfer is done without populating the excited states. In this paper we do not discuss gate operations, but we focus our attention on the structure of entangled dark states in systems with more than two states in both the atom and the cavity.

There is now a renewed interest in collective quantum states in the context of quantum information and quantum computing [8, 10, 11]. However, the focus of attention has shifted to the exact nature of the entangled state of the collection and the robustness of the state to decoherence. Theoretical and experimental work on multipartite entangled two-level quantum systems is currently the focus of research [12–15]. Although there is still much to be understood in these systems, especially on the theoretical aspect of classifying the types of entangled states in multipartite systems, there are also some work done in multipartite systems involving atoms that go beyond two levels [8, 13, 16]. The motivation of these works range from simply characterizing multipartite multilevel entangled systems to utilizing these systems for quantum memory. The purpose of our paper is to present the structure of the collective entangled dark states in a multipartite system, and to offer a general method to obtain these states for various atomic energy structures.

2. Collective entangled dark states

2.1 *Entangled dark state in $J_{1/2} - J_{1/2}$ system*

Here we will present a method to obtain the complete set of collective entangled dark states. First, we will consider a simple case of two $J_{1/2} - J_{1/2}$ systems interacting with three cavity fields with a total of one excitation, and then present the general method to obtain these collective dark states with N atoms.

The system we are considering consists of two atoms each with energy structure, as shown in figure 1, in a cavity with a single excitation shared among three modes.

The Hamiltonian of the system in the interaction picture is

$$\hat{H}_I = \hbar \sum_{n=1}^{2} \Big[g_1 \hat{\sigma}_{13}^{(n)} \hat{a}_1^\dagger + g_2 \hat{\sigma}_{14}^{(n)} \hat{a}_3^\dagger + g_3 \hat{\sigma}_{23}^{(n)} \hat{a}_2^\dagger + g_4 \hat{\sigma}_{24}^{(n)} \hat{a}_1^\dagger + H.C. \Big], \tag{2}$$

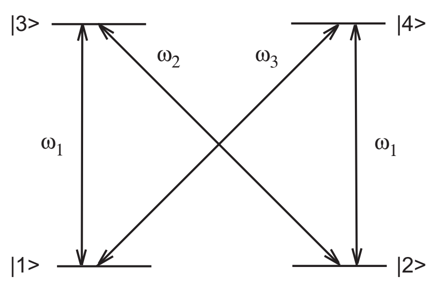

Figure 1. The energy diagram of the $J_{1/2} - J_{1/2}$ system.

where $\hat{\sigma}_{ij}^{(n)}$ is the atomic transition operator for the nth atom and $\hat{a}^\dagger$ is the field creation operator. For the purpose of this calculation we will assume the coupling constant g_i is the same for both atoms and all four transitions. The general single-excitation state of the system is

$$\begin{aligned}|\Psi\rangle = {} & c_1(t)|11;100\rangle + c_2(t)|11;010\rangle + c_3(t)|11;001\rangle + c_4(t)|22;100\rangle + c_5(t)|22;010\rangle \\ & + c_6(t)|22;001\rangle + c_7(t)|31;000\rangle + c_8(t)|13;000\rangle + c_9(t)|32;000\rangle + c_{10}(t)|23;000\rangle \\ & + c_{11}(t)|41;000\rangle + c_{12}(t)|14;000\rangle + c_{13}(t)|42;000\rangle + c_{14}(t)|24;000\rangle + c_{15}(t)|12;100\rangle \\ & + c_{16}(t)|12;010\rangle + c_{17}(t)|12;001\rangle + c_{18}(t)|21;100\rangle + c_{19}(t)|21;010\rangle + c_{20}(t)|21;001\rangle,\end{aligned} \tag{3}$$

where first two indices in the ket are the atomic states, and the three indices following the semicolon are the number of photons in the different field modes corresponding to ω_1, ω_2 and ω_3.

We note that in this physical system there is permutation symmetry between the two atoms and a symmetry in the exchange of energy levels $|1\rangle$, $|3\rangle$ and $|2\rangle$, $|4\rangle$. That is to say that the interaction Hamiltonian is invariant under the switching of the atoms and switching between 1 and 2 index with the 3 and 4 index, respectively. This suggests that we can switch the basis we work in to the symmetric and antisymmetric linear combination of both the permutated atomic states and the switched energy states to block diagonalize the interaction Hamiltonian. This new basis we work in is

$$\begin{aligned} & (++) \\ b_1 &: |11;100\rangle + |22;100\rangle \\ b_2 &: |12;010\rangle + |21;001\rangle + |21;010\rangle + |12;001\rangle \\ b_3 &: |13;000\rangle + |24;000\rangle + |31;000\rangle + |42;000\rangle \\ b_4 &: |11;001\rangle + |22;010\rangle \\ b_5 &: |12;100\rangle + |21;100\rangle \\ b_6 &: |23;000\rangle + |14;000\rangle + |32;000\rangle + |41;000\rangle \\ b_7 &: |11;010\rangle + |22;001\rangle, \end{aligned}$$

$$\begin{aligned} & (+-) \\ b_8 &: |32;000\rangle - |41;000\rangle + |23;000\rangle - |14;000\rangle \\ b_9 &: |11;001\rangle - |22;010\rangle \\ b_{10} &: |11;100\rangle - |22;100\rangle \\ b_{11} &: |12;010\rangle - |21;001\rangle + |21;010\rangle - |12;001\rangle \\ b_{12} &: |13;000\rangle - |24;000\rangle + |31;000\rangle - |42;000\rangle \\ b_{13} &: |11;010\rangle - |22;001\rangle, \end{aligned}$$

$$
\begin{aligned}
&(-+)\\
&b_{14}:|23;000\rangle+|14;000\rangle-|32;000\rangle-|41;000\rangle\\
&b_{15}:|12;010\rangle+|21;001\rangle-|21;010\rangle-|12;001\rangle\\
&b_{16}:|13;000\rangle+|24;000\rangle-|31;000\rangle-|42;000\rangle,\\
&(--)\\
&b_{17}:|12;100\rangle-|21;100\rangle\\
&b_{18}:|32;000\rangle-|41;001\rangle-|23;000\rangle+|14;000\rangle\\
&b_{19}:|13;000\rangle-|24;000\rangle-|31;010\rangle+|42;000\rangle\\
&b_{20}:|12;010\rangle-|21;001\rangle-|21;010\rangle-|12;001\rangle.
\end{aligned}
$$

Here the $(+-)$ denotes those states that are symmetric in the exchange of atoms and antisymmetric in the switching of the levels, $(++)$ denotes states that are symmetric in both the exchange of atoms and the switching of the levels, and so forth. In this basis the interaction Hamiltonian turns block diagonal, making it far more manageable than the 20×20 matrix we had before. The blocks of the interaction Hamiltonian in the new basis are

$$
H_{1,2,3}=g\begin{pmatrix}0&0&2^{1/2}\\0&0&1\\2^{1/2}&1&0\end{pmatrix},\quad H_{4,5,6}=g\begin{pmatrix}0&0&2^{1/2}\\0&0&2^{1/2}\\2^{1/2}&2^{1/2}&0\end{pmatrix},\quad H_{8,9}=g\begin{pmatrix}0&-2^{1/2}\\-2^{1/2}&0\end{pmatrix},
$$

$$
H_{10,11,12}=g\begin{pmatrix}0&0&2^{1/2}\\0&0&1\\2^{1/2}&1&0\end{pmatrix},\quad H_{15,16}=g\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad H_{17,18}=g\begin{pmatrix}0&2^{1/2}\\2^{1/2}&0\end{pmatrix},
$$

$$
H_{19,20}=g\begin{pmatrix}0&1\\1&0\end{pmatrix}. \tag{4}
$$

The matrix H_{ijk} denotes the block of the interaction Hamiltonian in which the columns of the matrix represent the bases b_i, b_j, b_k. The largest block in this matrix is 3×3, so it is easy to diagonalize each block to find the dark states of this system, i.e. eigenstates of the Hamiltonian which cannot spontaneously emit because they are linear combinations of ground states only. These dark states of this system are

$$
\begin{aligned}
|\Psi_{\mathrm{D1}}\rangle&=\frac{1}{3^{1/2}}\Big[|11;100\rangle-|12;010\rangle-|21;010\rangle\Big],\\
|\Psi_{\mathrm{D2}}\rangle&=\frac{1}{3^{1/2}}\Big[|22;100\rangle-|12;001\rangle-|21;001\rangle\Big],\\
|\Psi_{\mathrm{D3}}\rangle&=\frac{1}{2}\Big[|11;001\rangle-|21;100\rangle-|12;100\rangle+|22;010\rangle\Big].
\end{aligned}\tag{5}
$$

Note that the dark states of this system are entangled in both the atoms and the field. And they are not due to simple cancellation of the dipole moments between the two atoms. Also, because these states only contain ground states of the atoms, they are resistant to spontaneous decay. It is easy to see that these dark states are also eigenstates of the interaction Hamiltonian containing the spontaneous decay term,

$$\hat{H}_I = \hbar \sum_{n=1}^{2} \Big[g_1 \hat{\sigma}_{13}^{(n)} \hat{a}_1^\dagger + g_2 \hat{\sigma}_{14}^{(n)} \hat{a}_3^\dagger + g_3 \hat{\sigma}_{23}^{(n)} \hat{a}_2^\dagger + g_4 \hat{\sigma}_{24}^{(n)} \hat{a}_1^\dagger + \sum_k \Big(g_k \hat{b}_k^\dagger (\hat{\sigma}_{13}^{(n)} + \hat{\sigma}_{23}^{(n)} + \hat{\sigma}_{14}^{(n)} + \hat{\sigma}_{24}^{(n)}) \Big) + H.C. \Big]. \tag{6}$$

2.2 *General approach for obtaining collective entangled dark states*

The approach taken for a general N-atom case is similar to the bipartite case we have just discussed. We first identify the essential states of the wave function and then change basis to one that is analogous to the symmetric and antisymmetric linear combinations of these bases to block diagonalize the Hamiltonian.

For N-atoms of the $J_{1/2}$ to $J_{1/2}$ system with one photon in the cavity there are

$$3\left[1 + \sum_{k=1}^{N} 2^{k-1}\right] = 3 \cdot 2^N \tag{7}$$

states whose indices contain only the atomic ground states, and there are

$$2N\left[1 + \sum_{k=1}^{N-1} 2^{k-1}\right] = N \cdot 2^N \tag{8}$$

states which contains one atom in the collection to be excited. The dimensionality of the system is obtained by adding these two equations

$$N \cdot 2^N + 3 \cdot 2^N = 2^N \Big[N + 3\Big]. \tag{9}$$

Now we change basis to a Fourier basis with respect to the permutation of the atoms which is given by

$$f_j \colon \sum_{k=1}^{N} \exp\left(\mathrm{i}k \frac{2\pi}{N} \right) \big| i_1^k, i_2^{k+1}, \ldots, i_N^{k+N} \big\rangle. \tag{10}$$

The i_p^q indicates the pth index of the essential basis state shifted to the qth position. The q index is modulo $(N+1)$ which is to say the last index shifts to the first position. Similarly, we perform the same operation for the exchange of the atomic energy levels,

$$h_i \colon \sum_{k=1}^{S} \exp\left(\mathrm{i}k \frac{2\pi}{S} \right) | f_j^S \rangle. \tag{11}$$

Here S is the number of energy exchange symmetry the atom contains. In our example of $J_{1/2} \to J_{1/2}$ system $S = 2$, so we get the familiar symmetric/antisymmetric linear combinations with respect to the energy levels. Also note that the states in the sum are the Fourier basis states of the atomic permutation, $|f_j^S\rangle$, where the superscript S denotes Sth energy exchange symmetry of the state $|f_j\rangle$.

Once the Hamiltonian is put in the h_i basis the Hamiltonian would be block diagonal, and one can diagonalize the blocks relatively easily. The procedure can be applied to any system to turn the Hamiltonian into block diagonal form. However, the size of the blocks will be determined by the number of symmetries the system in question contains. The more the symmetry, the smaller the blocks will be.

3. Conclusion

In this paper we presented a general approach to obtaining the structure of the complete set of collective entangled states, and we also offer an example by applying the method to a $J_{1/2} \rightarrow J_{1/2}$ system. The purpose was to shed light on the nature of collective entangled states through gaining insight into the exact structure of these states.

References

[1] A. Beige, D. Braun and P.L. Knight, New J. Phys. **2** 22 (2000).
[2] R.H. Dicke, Phys. Rev. **93** 99 (1954).
[3] A. Crubellier, S. Liberman, D. Pavolini, *et al.*, J. Phys. B: At. Mol. Phys. **18** 3811 (1985).
[4] R. Friedberg, S.R. Hartmann and J.T. Manassah, Phys. Lett. A **40** 365 (1972).
[5] R.N. Deb, M.S. Abdalla, S.S. Hassan, *et al.*, Phys. Rev. A **73** 053817 (2006).
[6] G.S. Agarwal and R.R. Puri, Phys. Rev. A **49** 4968 (1994).
[7] A. Kuzmich, L. Mandel and N.P. Bigelow, Phys. Rev. Lett. **85** 1594 (2000).
[8] M. Fleischhauer, S.F. Yelin and M. Lukin, Opt. Commun. **179** 395 (2000).
[9] T. Pellizzari, S.A. Gardiner, J.I. Cirac, *et al.*, Phys. Rev. Lett. **75** 3778 (1995).
[10] R. Raussendorf and H.J. Briegel, Phys. Rev. Lett. **86** 5188 (2001).
[11] M.A. Nielsen, Phys. Rev. Lett. **93** 040503 (2004).
[12] D. Bruß, N. Datta, A. Ekert, *et al.*, Phys. Rev. A **72** 014301 (2005).
[13] E. Solano, G.S. Agarwal and H. Walther, Phys. Rev. Lett. **90** 027903 (2003).
[14] H. Mikami, Y. Li and T. Kobayashi, Phys. Rev. A **70** 052308 (2004).
[15] M.O. Scully, E.S. Fry, C.H.R. Ooi, *et al.*, Phys. Rev. Lett. **96** 010501 (2006).
[16] A. Miyake, Phys. Rev. A **69** 012101 (2004).