

Deterministic secure communications using two-mode squeezed states

Alberto M. Marino* and C. R. Stroud, Jr.

The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

(Received 15 May 2006; published 18 August 2006)

We propose a scheme for quantum cryptography that uses the squeezing phase of a two-mode squeezed state to transmit information securely between two parties. The basic principle behind this scheme is the fact that each mode of the squeezed field by itself does not contain any information regarding the squeezing phase. The squeezing phase can only be obtained through a joint measurement of the two modes. This, combined with the fact that it is possible to perform remote squeezing measurements, makes it possible to implement a secure quantum communication scheme in which a deterministic signal can be transmitted directly between two parties while the encryption is done automatically by the quantum correlations present in the two-mode squeezed state.

DOI: [10.1103/PhysRevA.74.022315](https://doi.org/10.1103/PhysRevA.74.022315)

PACS number(s): 03.67.Hk, 03.67.Dd, 03.67.Mn, 42.50.Dv

One of the most interesting applications to have emerged from the recent growth in quantum optics is quantum cryptography. This field started with the pioneering work of Bennett and Brassard [1]. The basic idea is to use unique quantum-mechanical properties, such as entanglement and superposition of states, in order to transmit a signal securely between two parties. These fundamental quantum-mechanical properties allow us, in principle, to detect the presence of an eavesdropper with certainty, thus making it possible to determine if a channel is secure. The basic mechanism used in quantum cryptography for the detection of an eavesdropper is that any measurement inevitably disturbs the state of the system.

All of the initial quantum cryptography schemes were based either on single photons [1] or entangled photon pairs [2]. As a result, their experimental implementation is limited due to the lack of good single-photon sources as well as efficient single-photon detectors. In order to overcome these limitations there have been some recent proposals that extend the ideas of discrete quantum cryptography into the domain of continuous variables (CVs) [3–9]. For these new schemes, the two-mode squeezed state (TMSS) has proven to be the basic source of continuous variable entanglement. These quantum states of the field are routinely generated, the most common method being the optical parametric oscillator (OPO) [10,11], and can be detected with high efficiency through the use of homodyne detection.

In this paper we propose a CV quantum cryptography scheme that uses the squeezing phase of a TMSS in order to transmit information between two parties. The scheme is based on the facts that each mode of a TMSS by itself does not contain any information regarding the squeezing phase, and that it is possible to perform remote squeezing measurements. As opposed to most previous schemes, the scheme presented here allows for the secure transmission of a deterministic signal. It thus allows for the secure transmission of an encryption key as well as the possibility of sending a message directly. In the case in which a message is transmitted directly, the encryption is done automatically by the

quantum correlations present between the modes of a TMSS. As with any other quantum communication protocol, it is necessary to verify the security of the quantum channel used. This can be done, as well, with the help of the quantum correlations present in a TMSS [12,13].

We start by looking at the relevant properties of a TMSS. The two-mode squeezed state is defined according to [14]

$$|\alpha, \beta; \zeta\rangle = \hat{D}_1(\alpha)\hat{D}_2(\beta)\hat{S}_{12}(\zeta)|0, 0\rangle, \quad (1)$$

where $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the displacement operator, $\hat{S}_{12}(\zeta) = \exp(\zeta^*\hat{a}_1\hat{a}_2 - \zeta\hat{a}_1^\dagger\hat{a}_2^\dagger)$ is the two-mode squeezing operator, and $\zeta = se^{i\theta}$ is the squeezing parameter. The quantum correlations present in a TMSS become more evident when it is written in terms of number states. Such an expansion can be shown to be given by [15]

$$|\alpha, \beta; \zeta\rangle = \frac{1}{\cosh s} \sum_{n=0}^{\infty} (-e^{i\theta} \tanh s)^n |\alpha; n\rangle |\beta; n\rangle, \quad (2)$$

where s is the degree of squeezing, θ is the squeezing phase, and $|\alpha; n\rangle = \hat{D}(\alpha)|n\rangle$ is the displaced number state. As we can see from Eq. (2), there are perfect quantum correlations between the displaced number states of the two modes.

We now investigate the properties of the individual modes through the reduced density matrices, which from Eq. (2) can be shown to be given by

$$\hat{\rho}_1 = \frac{1}{\cosh^2 s} \sum_{n=0}^{\infty} (\tanh s)^{2n} |\alpha; n\rangle \langle \alpha; n|,$$

$$\hat{\rho}_2 = \frac{1}{\cosh^2 s} \sum_{n=0}^{\infty} (\tanh s)^{2n} |\beta; n\rangle \langle \beta; n|. \quad (3)$$

The main thing to note from Eq. (3) is that each mode by itself carries no information regarding the squeezing phase, only about the degree of squeezing. As a result, if one looks at the noise of an individual mode using homodyne detection, it will be phase independent. It is necessary to make a combined measurement of the two modes in order to extract the squeezing phase. It is then possible to take advantage of

*Electronic address: marino@optics.rochester.edu

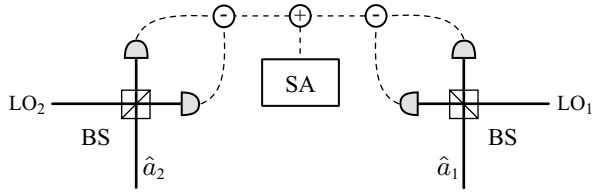


FIG. 1. Detection scheme for remote squeezing measurements. A local homodyne measurement of each mode is performed, after which the resulting photocurrents are combined to obtain a complete characterization of the two-mode squeezed state. Operators \hat{a}_1 and \hat{a}_2 represent the two modes of the squeezed state. Notation: LO=local oscillator; BS=beam splitter; SA=spectrum analyzer.

this property of a TMSS and use the squeezing phase to encode information. In fact, each mode by itself has more noise than a coherent state; however, due to the fact that the noise in the two modes is correlated it is possible to get the noise below that of a coherent state when making a joint measurement. This fact was used by Pereira *et al.* [9] in order to hide a classical message in the excess noise of each mode and then use the quantum correlations between the modes to extract the message.

In order to take advantage of the fact that the squeezing phase can only be obtained from a combined measurement of both modes of the squeezed field, it is necessary to use a measurement technique that allows us to perform a remote squeezing measurement. This can be done using the scheme shown in Fig. 1 [16]. By using this detection scheme it is possible to perform local balanced homodyne measurements on each of the modes of the TMSS and then combine the resulting measurements in order to extract the squeezing phase information. Once the two local measurements are combined, the variance of the resulting signal can be shown to be of the form

$$\langle (\Delta \hat{n}_{12})^2 \rangle = 2|E_{LO}|^2 \left[e^{-2s} \cos^2 \left(\frac{\chi_1 + \chi_2 - \theta}{2} \right) + e^{2s} \sin^2 \left(\frac{\chi_1 + \chi_2 - \theta}{2} \right) \right], \quad (4)$$

where E_{LO} is the amplitude of the local oscillator (LO), χ_1 is the relative phase between LO1 and the corresponding mode of the TMSS, \hat{a}_1 , and χ_2 is the relative phase between LO2 and \hat{a}_2 . In deriving this expression it has been assumed that the amplitude of the two local oscillators is the same. As can be seen from Eq. (4), it is possible to change the quadrature that is measured by changing the phase of either LO.

Using these two elements it is possible to implement a quantum cryptography scheme that uses the squeezing phase of a TMSS to transmit information between two parties, Alice and Bob. The basic communication scheme is shown in Fig. 2. Alice uses a squeezed light source, such as an OPO, to generate a TMSS and sends one of the modes to Bob over a quantum channel while keeping the other mode. In order for Bob to perform his homodyne measurements, he needs to have a LO which is phase coherent with the mode he receives. Alice can also send a coherent state LO to Bob by combining it with the squeezed beam by using a polarizing

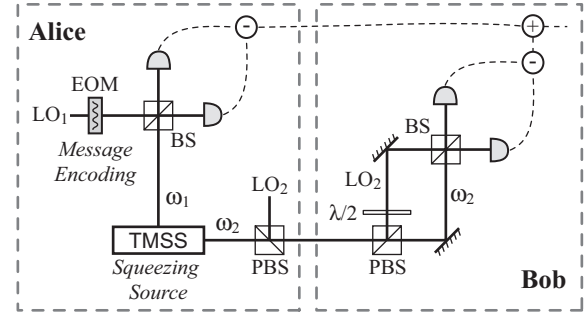


FIG. 2. Proposed quantum cryptography scheme. The squeezing phase of the TMSS is used to transmit information between Alice and Bob. The information is encoded through the modulation of the local oscillator used by Alice to perform her homodyne measurements. The information is decoded by Bob by combining his and Alice's measurement results. Notation: TMSS=two-mode squeezed state; LO=local oscillator; EOM=electro-optical modulator; BS=beam splitter; PBS=polarizing beam splitter; $\lambda/2$ =half wave plate.

beam splitter (PBS). Bob then splits the LO and the squeezed mode using another PBS and performs a homodyne measurement. When performing his measurements, Bob keeps the phase of the LO constant. As can be seen from the reduced density matrix of the TMSS, Eq. (3), Bob will have gained no information, except for the degree of squeezing, by performing this measurement. Which means that if an eavesdropper, Eve, were to intercept the mode sent to Bob, she would not be able to extract any information either.

As can be seen from Eq. (4), the noise of the combined homodyne measurements of the modes of the TMSS can be switched between the minimum and maximum noise levels by performing a π phase shift on the LO of either Alice or Bob. Thus in order to transmit the signal, Alice encodes the information on her part of the TMSS by changing the phase of the LO she uses to perform her homodyne measurements. This can easily be done with the help of an electro-optic modulator, thus allowing for a high speed encoding of the signal. Once Alice has encoded the information on her mode by making the homodyne measurements and Bob has finished making his set of homodyne measurements, she can send her measurement results to Bob over a public channel. As is the case with the mode sent to Bob, the measurement results obtained by Alice will contain no information on the encoded signal. It is necessary to combine the two measurements in order to retrieve the signal. Once Bob receives the measurements from Alice he can combine both his and Alice's measurements to decode the signal. Since Bob keeps the phase of his LO constant, once he combines both measurements, he will see a signal whose variance will be changing between two different levels which represent the information encoded by Alice.

As with any other quantum key distribution (QKD) scheme, it is necessary to verify the security of the quantum channel used to send Bob his part of the TMSS. In the proposed scheme described above, this can be done through the quantum correlations present in the TMSS. Since Bob does not need to change the phase of his LO when performing his measurements, all the information he decodes after combin-

ing both his and Alice's measurements can be used both for generating a cryptographic key and verifying the security of the channel. A complete security analysis of the proposed scheme is beyond the scope of this paper. Instead, we focus on the usual attack strategies for continuous variables [3,17], such as intercept and resend and partial interception of the mode sent to Bob.

Once the measurement is performed by Alice, the resulting photocurrent can be treated as a classical signal [18]. Thus as long as the electronic noise of the components used is low enough, any division or amplification of this classical signal will not affect the final result. In the following analysis we thus consider the worst case scenario, which corresponds to one in which Eve can make a perfect copy of the information sent through the public channel without modifying it.

The easiest form of attack is the intercept and resend strategy. In this case Eve intercepts the mode sent to Bob. As can be seen from Eq. (3), Eve will have gained no information on the squeezing phase from this mode and thus no information on the transmitted signal. Since in principle it is possible to extract the degree of squeezing present in the TMSS from the measurement of a single mode, Eve could resend a mode of a TMSS with the same degree of squeezing to Bob. However, the new mode sent out by Eve will not be entangled with the mode retained by Alice. Thus when Bob combines his and Alice's measurement results the combined signal will contain more noise than a coherent state and will be phase independent. That is, there will be no squeezing and the modulation of the variance will not be present. Thus this type of attack can easily be detected.

The other usual strategy of attack is partial interception. In this case Eve splits part of the signal and uses this together with the classical signal to gain information on the transmitted signal. To see the effect this type of attack would have on the degree of squeezing we need to look at the case in which only the mode sent to Bob experiences losses. If we assume that Eve intercepts a portion $1 - \eta$ of the mode sent to Bob, then the variance of the combined signal takes the form

$$\begin{aligned} \langle (\Delta \hat{n}_{12})^2 \rangle = & |E_{LO}|^2 \left\{ (1 - \eta) + e^{-2s} \left[\frac{1 + \eta}{2} \right. \right. \\ & \left. \left. + \sqrt{\eta} \cos(\chi_1 + \chi_2 - \theta) \right] \right. \\ & \left. + e^{2s} \left[\frac{1 + \eta}{2} - \sqrt{\eta} \cos(\chi_1 + \chi_2 - \theta) \right] \right\}. \quad (5) \end{aligned}$$

In the case of a coherent state, $s=0$, Eq. (5) can be shown to be phase independent and to reduce to the expected level of $2|E_{LO}|^2$. In order to see the effect of partial interception on the amount of squeezing that is measured, we look at the measured degree of squeezing, which is defined according to

$$D(\eta) = 10 \log_{10} \left[\frac{\langle (\Delta \hat{n}_{12})^2 \rangle_{\min}}{\langle (\Delta \hat{n}_{12})^2 \rangle_{cs}} \right], \quad (6)$$

where $\langle (\Delta \hat{n}_{12})^2 \rangle_{\min}$ is the minimum variance of the squeezed state and $\langle (\Delta \hat{n}_{12})^2 \rangle_{cs}$ represents the variance of a coherent

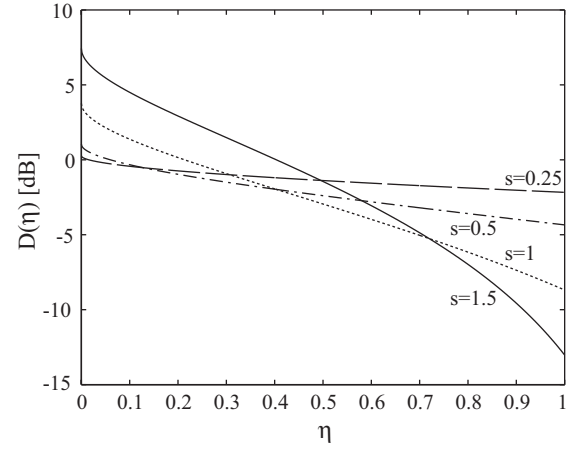


FIG. 3. Degree of squeezing measured by Bob after combining his and Alice's measurements as a function of the losses, $1 - \eta$, in the mode sent to Bob.

state. This quantity is plotted for different initial degrees of squeezing in Fig. 3. As can be seen from this figure any loss will result in a reduction of the measured degree of squeezing, as expected. The rate at which this happens becomes larger for a higher initial degree of squeezing, which means that the losses become more evident. Thus the ability to detect any eavesdropping increases as the initial squeezing parameter s increases. An interesting thing to notice from Fig. 3 is that as $\eta \rightarrow 0$ the noise goes above that of the coherent state. The reason for this is that each mode by itself has excess noise, so that if there is loss in only one of the modes then the cancellation of this excess noise is not perfect, thus leading to more noise than a coherent state for large losses.

If the minimum and maximum values of the variance, Eq. (5), are used for the transmission of the signal, then the signal-to-noise ratio (SNR) is given by

$$\begin{aligned} (\text{SNR}) &= \frac{\langle (\Delta \hat{n}_{12})^2 \rangle_{\max} - \langle (\Delta \hat{n}_{12})^2 \rangle_{\min}}{\langle (\Delta \hat{n}_{12})^2 \rangle_{\min}} \\ &= \frac{4\sqrt{\eta} \sinh 2s}{(1 - \eta) + e^{-2s}[(1 + \eta)/2 + \sqrt{\eta}] + e^{2s}[(1 + \eta)/2 - \sqrt{\eta}]}. \quad (7) \end{aligned}$$

The effect of losses on the SNR can be seen in Fig. 4. We again see that a higher degree of squeezing will lead to a larger change in SNR for a given amount of loss. From these results we have, for example, that for $s=1$, which corresponds to squeezing of around 8.5 dB, a reduction of the amount of squeezing measured and of the SNR of about 1 dB requires the interception of only 7% of the mode. Thus making it a very sensitive way of detecting any interception of part of the mode sent to Bob.

Eve could instead use a quantum attack, such as an optical tap [19] or a quantum nondemolition measurement [20] in order to gain some extra information without being detected. However, this type of attack would lead to increased noise in the quadrature conjugate to the quadrature that is measured. Since the proposed scheme effectively measures both

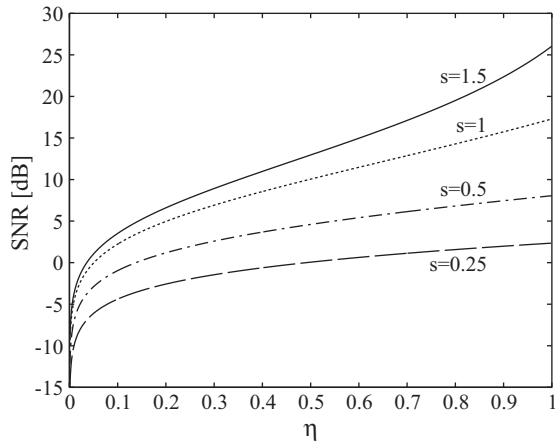


FIG. 4. Signal-to-noise ratio of quantum cryptography scheme as a function of the losses, $1 - \eta$, present in the mode of the TMSS sent to Bob.

quadratures, this type of attack can also be identified by monitoring the quantum correlations of the TMSS. By also modulating the squeezing phase θ , in addition to Alice's LO, an extra degree of security can be added for the quantum attacks described above. As can be seen from Eq. (4), as long as the quantity $\chi_1 - \theta$ is changed by a factor of π the scheme presented above will not be modified.

In practice, the degree of squeezing or correlations for a given quantum channel can be established. Thus once the losses in the quantum channel have been accounted for, both the degree of squeezing and the SNR can be used to verify if unauthorized access to the quantum channel has occurred.

As opposed to most previous schemes [3–8], which rely on both Alice and Bob making random measurements of the quadratures of the field, the scheme presented here does not require Bob to actively select between different measurements. Only a single beam, Alice's LO, needs to be phase modulated in order to encode the signal. As a result, a higher transmission rate can be achieved due to the fact that all the information received by Bob can be used both to generate the encryption key and to verify the security of the quantum channel through the quantum correlations present in the TMSS.

The fact that a deterministic signal can be sent with our scheme opens up the possibility of using it for the secure transmission of a message. This allows the bypassing of the encryption-decryption process, thus making the communication process more efficient while maintaining the security of the proposed continuous variable QKD scheme. When the scheme is used for the transmission of a cryptographic key, if the presence of an eavesdropper is detected then the whole key can be thrown away and a new one generated. However, this is not the case when a message is sent. In this case, in order to verify the security of the channel, Alice needs to

insert check bits at random times throughout the message. Since the check bits are inserted at random times, Eve will not know when to perform a measurement in order to intercept the message but not the check bits. As a result, the eavesdropping will affect both the message and the check bits. Once Bob measures the mode he received, Alice sends her measurements results on the check bits and tells Bob which time slots correspond to these bits. Bob can then use the information obtained from combining his and Alice's results for the check bits in order to verify the security of the quantum channel as described above. Once the security of the channel has been verified, Alice can send her measurement results for the actual message to Bob, who just needs to combine them with his results in order to decode the message. If the presence of an eavesdropper is detected, a different quantum channel needs to be used and the process repeated.

An important thing to consider for the proposed scheme is that in order to use the squeezing phase as the means to transmit information it is necessary to have a good control over the relative phase differences, χ_1 and χ_2 , between the local oscillators and the corresponding mode of the TMSS when performing the local homodyne measurements. In general, things such as the transmission of the LO and the mode of the TMSS from Alice to Bob will introduce phase fluctuations between the two fields. It is thus necessary to compensate for any phase fluctuations by actively stabilizing the relative phases when performing the homodyne measurements. It is possible to do such an active stabilization due to the fact that the variance of the combined local measurements, Eq. (4), depends on the relative phase difference between each of the local oscillators and the corresponding mode of the TMSS. Thus the beat note from each of the local homodyne measurements will contain information regarding the corresponding relative phase difference. This information can then be used to actively compensate for any phase fluctuations that may arise. The possibility of stabilizing the relative phase between the LO and the squeezed field when making a homodyne detection has been shown by Chelkowski *et al.* [21] for the case of a bright squeezed field and by McKenzie *et al.* [22] for the case of a vacuum squeezed state.

In conclusion, we have presented a quantum cryptography scheme that relies on the squeezing phase of a TMSS for the secure transmission of information. The security of the quantum channel used can be verified with the help of the quantum correlations present in the squeezed field, since the presence of an eavesdropper will result in a decrease in the degree of squeezing measured as well as an increase in the error rate of the transmitted signal. Apart from the usual application of quantum cryptographic systems for the transmission of encryption keys, the proposed scheme allows for the transmission of a deterministic signal between two parties, thus opening the possibility for direct transmission of messages.

- [1] C. H. Bennett and G. Brassard, in *Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India, 1984*, p. 175.
- [2] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
- [3] T. C. Ralph, Phys. Rev. A **61**, 010303(R) (1999).
- [4] M. D. Reid, Phys. Rev. A **62**, 062308 (2000).
- [5] M. Hillery, Phys. Rev. A **61**, 022309 (2000).
- [6] D. Gottesman and J. Preskill, Phys. Rev. A **63**, 022309 (2001).
- [7] C. Silberhorn, N. Korolkova, and G. Leuchs, Phys. Rev. Lett. **88**, 167902 (2002).
- [8] A. C. Funk and M. G. Raymer, Phys. Rev. A **65**, 042307 (2002).
- [9] S. F. Pereira, Z. Y. Ou, and H. J. Kimble, Phys. Rev. A **62**, 042311 (2000).
- [10] L. A. Wu, X. A. Min, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1465 (1987).
- [11] P. K. Lam, T. C. Ralph, B. C. Buchler, D. E. McClelland, H. A. Bachor, and J. Gao, J. Opt. B: Quantum Semiclassical Opt. **1**, 469 (1999).
- [12] M. D. Reid, Phys. Rev. A **40**, 913 (1989).
- [13] S. M. Barnett and S. J. D. Phoenix, Phys. Rev. A **44**, 535 (1991).
- [14] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).
- [15] S. M. Barnett and P. L. Knight, J. Mod. Opt. **34**, 841 (1987).
- [16] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, Phys. Rev. Lett. **68**, 3663 (1992).
- [17] T. C. Ralph, Phys. Rev. A **62**, 062306 (2000).
- [18] H. A. Bachor and T. C. Ralph, *A Guide to Experiments in Quantum Optics*, second edition (Wiley-VCH, Berlin, 2004).
- [19] J. P. Poizat and P. Grangier, Phys. Rev. Lett. **70**, 271 (1993).
- [20] P. Grangier, J. A. Levenson, and J. P. Poizat, Nature (London) **396**, 537 (1998).
- [21] S. Chelkowski, H. Vahlbruch, B. Hage, A. Franzen, N. Lastzka, K. Danzmann, and R. Schnabel, Phys. Rev. A **71**, 013806 (2005).
- [22] K. McKenzie, E. E. Mikhailov, K. Goda, P. K. Lam, N. Grosse, M. B. Gray, N. Mavalvala, and D. E. McClelland, J. Opt. B: Quantum Semiclassical Opt. **7**, S421 (2005).