Enhanced self-action effects by electromagnetically induced transparency in the two-level atom

Ryan S. Bennink, Robert W. Boyd, C. R. Stroud, Jr., and Vincent Wong
Institute of Optics, University of Rochester, New York 14627

(Received 9 June 2000; published 6 February 2001)

Electromagnetically induced transparency (EIT) has been studied primarily within the context of multilevel atomic systems. We show that EIT can occur also in a two-level atomic system and can lead to strong self-action and slow-light effects that are not hampered by material absorption, with important potential implications for processes such as squeezed-light generation and the propagation of optical solitons.

DOI: 10.1103/PhysRevA.63.033804 PACS number(s): 42.50.Gy, 32.80.−t, 42.50.Hz

Electromagnetically induced transparency (EIT) is a powerful technique that can be used to render a material system transparent to resonant laser radiation, while retaining the large and desirable nonlinear optical properties associated with the resonant response of a material system [1]. EIT has been observed in several different experimental configurations [2,3], and its occurrence in still other configurations has been predicted theoretically [4]. Laboratory studies have confirmed that EIT can be used to enhance the efficiency of physical processes including nonlinear frequency conversion [3,5] and optical phase conjugation [6]. In addition, it has been predicted that EIT can enhance the properties of a much broader range of processes including squeezed-light generation [7] and low-light-level photonic switching [8]. EIT has been observed both in atomic vapors [2,3,5,6,9] and in solids [10], and it plays a key role in the generation of slow light [11] and its concomitant production of extremely large nonlinear optical effects [12]. Intimately related to EIT is coherent population trapping [13], and the establishment of high refractive indices [14].

Most previous work on EIT has dealt with multilevel atomic systems. In this article, we present a theoretical analysis of EIT effects in the two-level atomic system. The analysis of EIT within the context of a much simpler level scheme leads to additional insight into the origin of EIT. Our analysis confirms previous work showing that the presence of a strong control field can establish frequencies at which the atomic absorption vanishes or is negative, and, most importantly, that the third-order nonlinear optical response leading to self-action effects can be quite large at these particular frequencies [15]. Thus two-level-atom EIT effects hold considerable promise for applications such as spatial soliton propagation [16], squeezed-light generation by self-phase modulation [19], and many types of optical switching devices [20]. We also find that low group velocities (slow light) can occur within the context of the strongly driven two-level atom, and that in some ways the large nonlinear optical response can be understood as a direct consequence of the slowness of the light propagation.

The origin of EIT in the two-level atom can be understood in terms of the energy-level diagram shown in Fig. 1, which shows a strong control field of frequency ω_c and amplitude E_c interacting with the atomic system. Population is coherently cycled between the lower level a and the upper level b at the Rabi frequency $\Omega' = (\Omega^2 + \Delta_c^2)^{1/2}$, where $\Omega = 2\mu_{ba}E_c/\hbar$, $\Delta_c = \omega_c - \omega_{ba}$, and ω_{ba} is the atomic transition frequency. In many ways, the strongly driven atomic system behaves as the four-level system also shown in the diagram, where the lower and upper bare atomic levels have split into doublets of separation $\hbar\Omega'$. This system consequently possesses three characteristic frequencies, ω_c and $\omega_c \pm \Omega'$. As is well known [21,22], and as can be seen from several of the examples shown below, two of these resonance frequencies correspond to gain features in the response experienced by an additional signal field also applied to the strongly driven two-level atom. Since these gain features adjoin regions of normal absorption, there are several (in fact, as many as four) frequencies at which the signal-wave absorption precisely vanishes. For these frequencies the two-level system displays EIT. The vanishing of the absorption can be understood as a consequence of quantum interference between various pathways shown in Fig. 1.

Our mathematical analysis proceeds by calculating the response of a two-level atomic system to an applied field of the form $\tilde{E}(t) = E_c \exp(-i\omega_c t) + E_s \exp(-i\omega_s t) + \text{c.c.}$, where E_c

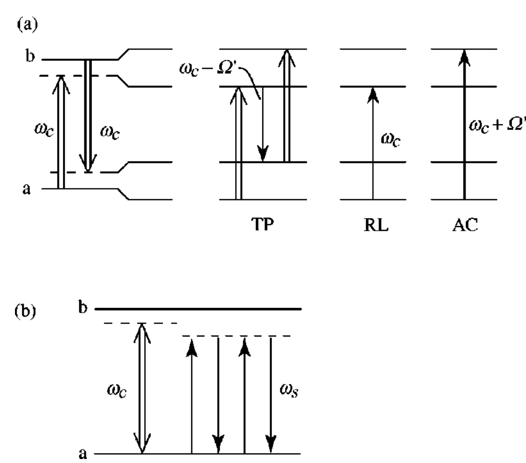

FIG. 1. (a) Origin of EIT in the two-level atom. In the presence of the strong control field, the lower and upper levels split into doublets with separation equal to the Rabi frequency Ω', and consequently the dressed atom possesses the three resonance frequencies that are shown. Two of these features can give rise to amplication of a signal wave as a consequence of the three-photon effect (TP) or to stimulated Rayleigh scattering (RL). AC denotes the ac-Stark-shifted atomic resonance. (b) The presence of the control field ω_c can enhance the efficiency of the indicated four-wave-mixing process involving the signal field ω_s, as described in the text.

and E_s represent the amplitudes of the strong control field and weaker (but still strong) signal field, respectively. We take the density matrix equations of motion in the form [22]

$$\dot{p}=(I\delta_c-\gamma)p+\hbar^{-1}|\mu_{ba}|^2 w[E_c+E_s e^{-i(\omega_s-\omega_c)t}], \quad (1)$$

$$\dot{w}=-\Gamma(1+w)-\frac{4}{i\hbar}\,\mathrm{Im}\,p^*[E_c+E_s e^{-i(\omega_s-\omega_c)t}], \quad (2)$$

where $p=\rho_{ba}\mu_{ab}\exp(i\omega_c t)$ is the slowly varying dipole moment expectation value, $w=\rho_{bb}-\rho_{aa}$ is the inversion, Γ is the population decay rate, and γ is the dipole dephasing rate. We solve these equations correct to all orders in E_c and to third order in E_s to find the contributions $p_1^{(1)}$ and $p_1^{(3)}$ to the atomic dipole moment oscillating at frequency ω_s. These quantities are, respectively, the contributions linear and third-order in the amplitude E_s of the probe field. We express the results in terms of the linear susceptibility $\chi^{(1)}(\omega_s)$ and third-order susceptibility $\chi^{(3)}(\omega_s;\omega_s,\omega_s,-\omega_s)$; these quantities give the material response to the field at frequency ω_s as modified by the presence of the control field ω_c. We find that

$$\chi^{(1)}(\omega_s)=\frac{Np_1^{(1)}}{E_s}=\frac{2N|\mu_{ba}|^2}{\hbar\Gamma}\overline{\chi}^{(1)}, \quad (3)$$

$$\chi^{(3)}(\omega_s;\omega_s,\omega_s,-\omega_s)=\frac{Np_1^{(3)}}{E_s|E_s|^2}=\frac{8N|\mu_{ba}|^4}{(\hbar\Gamma)^3}\overline{\chi}^{(3)}, \quad (4)$$

where we have introduced the normalized, dimensionless susceptibilities

$$\overline{\chi}^{(1)}=\frac{(\overline{\Delta}^*-\delta)[(\delta+i)w_0^{(0)}-\Omega_c p_0^{(0)*}]+\frac{1}{2}|\Omega_c|^2 w_0^{(0)}}{2D(\tilde{\Delta},\delta)}, \quad (5)$$

$$\overline{\chi}^{(3)}=\frac{(\overline{\Delta}^*-\delta)[(\delta+i)\eta_0^{(2)}+\Omega_c(\overline{\chi}_2^{(2)}-\overline{\chi}_0^{(2)*})]+\frac{1}{2}(|\Omega_c|^2\eta_0^{(2)}-\Omega_c^2\eta_2^{(2)})}{2D(\tilde{\Delta},\delta)}. \quad (6)$$

These susceptibilities are expressed in terms of the quantities $\tilde{\Delta}=(\Delta_c+i\gamma)/\Gamma$, $\delta=(\omega_s-\omega_c)/\Gamma$, $\Omega_c=2\mu_{ba}E_c/\hbar\Gamma$, $\overline{\gamma}=\gamma/\Gamma$, $D(\tilde{\Delta},\delta)\equiv(\tilde{\Delta}+\delta)(\tilde{\Delta}^*-\delta)(\delta+i)+(\delta+i\overline{\gamma})|\Omega_c|^2$, and the auxiliary functions

$$\eta_0^{(2)}=\frac{\overline{\gamma}w_0^{(0)}-2\,\mathrm{Im}[\tilde{\Delta}(\delta+2i\overline{\gamma})\overline{\chi}^{(1)}]}{|\tilde{\Delta}|^2+\overline{\gamma}|\Omega_c|^2}, \quad (7)$$

$$\overline{\chi}_0^{(2)}=\frac{1}{2\tilde{\Delta}\Omega_c^*}[|\Omega_c|^2\eta_0^{(2)}-2(\tfrac{1}{2}w_0^{(0)}-(\tilde{\Delta}+\delta)\overline{\chi}^{(1)})^*], \quad (8)$$

$$\eta_2^{(2)}=\frac{\Omega_c^*}{\Omega_c}\,\frac{(3\delta+2i\overline{\gamma})(\tilde{\Delta}^*-2\delta)}{\tilde{\Delta}^*-\delta}\,\frac{[\frac{1}{2}w_0^{(0)}-(\tilde{\Delta}+\delta)\overline{\chi}^{(1)}]}{D(\tilde{\Delta},2\delta)}, \quad (9)$$

$$\overline{\chi}_2^{(2)}=-\frac{2(\tilde{\Delta}^*-\delta)(\tilde{\Delta}^*-2\delta)(2\delta+i)+\delta|\Omega_c|^2}{2\Omega_c(\tilde{\Delta}^*-\delta)}\times\frac{[\frac{1}{2}w_0^{(0)}-(\tilde{\Delta}+\delta)\overline{\chi}^{(1)}]}{D(\tilde{\Delta},2\delta)}, \quad (10)$$

$$w_0^{(0)}=-\frac{|\tilde{\Delta}|^2}{|\tilde{\Delta}|^2+\overline{\gamma}|\Omega_c|^2}, \quad (11)$$

$$p_0^{(0)}=-\frac{1}{2}\,\frac{\Omega_c\tilde{\Delta}^*}{|\tilde{\Delta}|^2+\overline{\gamma}|\Omega_c|^2}. \quad (12)$$

In terms of this model, we can also determine the inverse of the light group velocity $1/v_g\equiv n_g/c\equiv\beta_1=dk/d\omega$ and the group-velocity dispersion parameter $\beta_2=d^2k/d\omega^2$ through use of the relations $k=n\omega/c$ and $n\approx1+2\pi\chi^{(1)}$. In the figures below we plot scaled versions of the contribution per atom to these quantities, defined by

$$\overline{\beta_1}=d\chi^{(1)}/d\tilde{\Delta}_s, \quad \overline{\beta_2}=d^2\chi^{(1)}/d\tilde{\delta}_s^2. \quad (13)$$

Some of the predictions of this model are shown in Fig. 2. Part (a) of the figure shows the linear susceptibility (left column), nonlinear susceptibility (center column), and group velocity and its dispersion (right column) in the absence of the control field. These curves reproduce the well-known predictions of the optical properties of the two-level atom. Part (b) of the figure shows how these predictions are modified when a moderately strong, centrally tuned control field is simultaneously applied to the atom. We see that there are four frequencies at which Im $\chi^{(1)}$ vanishes identically. In addition, there are broad regions in which Im $\chi^{(1)}$ is negative, corresponding to signal-wave amplification. Under some circumstances, amplification may be desirable; under other circumstances ideal transparency may be preferable. The center

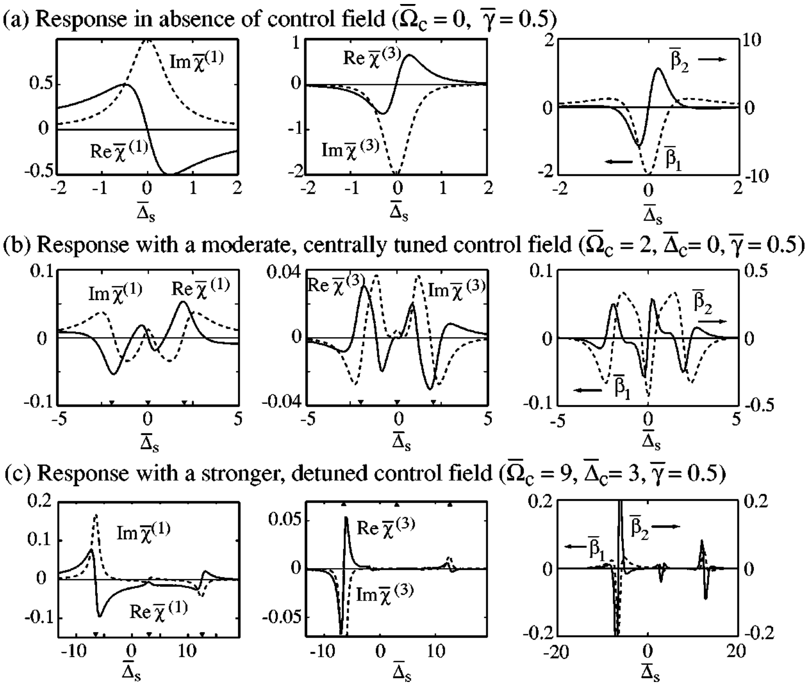

FIG. 2. Linear (left column) and nonlinear (center column) optical susceptibilities and group-velocity parameters (right column) experienced by the signal field (a) in the absence of the control field, (b) in the presence of a centrally tuned control field ($\Omega_c = 2$, $\overline{\Delta_c} \equiv \Delta_c/\Gamma = 0$), and (c) in the presence of a detuned control field ($\Omega_c = 9$, $\overline{\Delta_c} = 3$). Normalized susceptibilities defined by Eqs. (3) and (4) and normalized group velocity parameters defined by Eqs. (13) are shown. All cases assume radiative broadening $\overline{\gamma} \equiv \gamma/\Gamma = 0.5$, and the response is plotted as a function of $\overline{\Delta_s} = (\omega_s - \omega_{ba})/\Gamma$, where Γ is the population decay rate. The triangular markers in (b) and (c) show the locations of the three resonance frequencies defined in Fig. 1.

panel shows the real and imaginary parts of $\chi^{(3)}$. As can be seen from these graphs, or more explicitly from Fig. 3, the peaks of Re $\chi^{(3)}$ occur at exactly the frequencies at which both Im $\chi^{(1)}$ and Im $\chi^{(3)}$ vanish. Thus, this example illustrates essentially perfect conditions for the occurrence of EIT. Part (c) of Fig. 2 shows an example in which the control field is increased somewhat in intensity and is detuned from line center. In this situation the spectral features associated with each resonance of Fig. 1 are well separated, and it can be seen that the three-photon resonance and the stimulated Rayleigh resonance lead to gain features. We also see that in this case there are three frequencies for which Im $\chi^{(1)}$ vanishes. From the central panel of part (c) we see that Re $\chi^{(3)}$ is large at the frequencies at which Im $\chi^{(1)}$ is negative or vanishes. Of particular interest is the right-hand panel for each of these cases, which presents predictions for the group velocity and its dispersion. The group velocity can be either positive or negative with a magnitude either smaller or greater than that of the velocity of light in vacuum, *c*. The significance of superluminal [17] and backward [18] light propagation has been described previously. Also apparent in all cases is a

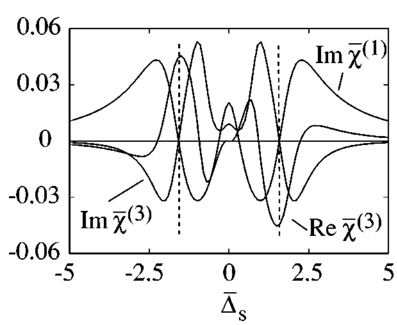

FIG. 3. Overlay of three curves from case (b) of Fig. 2. Note that both Im $\chi^{(1)}$ and Im $\chi^{(3)}$ vanish at the peaks of Re $\chi^{(3)}$.

tendency for the imaginary part (but not the real part) of $\chi^{(3)}$ to scale in proportion to β_1. A simplistic view of the relation between slow light and nonlinear optics holds that resonant media possess large nonlinear susceptibilities *because* the light propagates slowly and thus spends more time interacting with the material medium. The observation mentioned above shows that such an interpretation can be made only for the absorptive part of the nonlinear optical response. The relation between EIT and ultraslow light propagation has been elucidated in a much more detailed manner by Harris and Hau [12].

We next describe several possible applications of EIT effects in the two-level atom. In all of these examples, in order to make realistic predictions we include the effects of atomic motion by performing an average over a Maxwellian velocity distribution.

Figure 4 shows some predictions for a sodium atomic vapor at a number density of $1.2 \times 10^{15}\,\mathrm{cm}^{-3}$. We also assume that an argon buffer gas is present at a number density of $5 \times 10^{18}\,\mathrm{cm}^{-3}$, which broadens the atomic resonance to a width (full width at half maximum) of 600 MHz. The control laser is detuned 10.5 GHz to the high-frequency side of the atomic resonance, and has an intensity of 33 kW/cm^2, with a corresponding Rabi frequency of 16 GHz. We see that there is a frequency in the wings of the Stark-shifted atomic resonance at which Im $\chi^{(1)}$ vanishes but Re $\chi^{(3)}$ is large. We find that Re $\chi^{(3)} = 1.5 \times 10^{-7}$ esu at this frequency.

The efficiency of many nonlinear optical processes can be quantified in terms of the nonlinear phase shift imparted to the signal wave in passing through the interaction region. Let us estimate the value of this phase shift under the conditions of Fig. 4. Under conditions of EIT, the signal wave experiences no absorption, and thus the maximum nonlinear phase

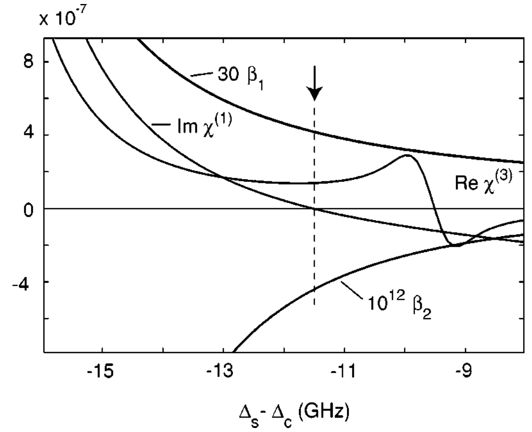

FIG. 4. Predictions of the optical response in the wing of the shifted atomic resonance. The influence of Doppler averaging is included in these predictions. Note that conditions exist for the establishment of spatiotemporal solitons. $\chi^{(3)}$ is given in esu, β_1 in $\mathrm{s\,m^{-1}}$, and β_2 in $\mathrm{s^2\,m^{-1}}$.

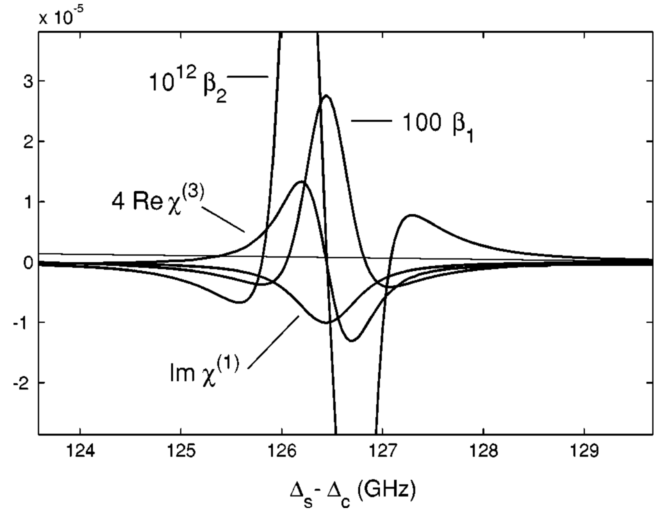

FIG. 5. Prediction of the optical response for a spectral region near the three-photon resonance. $\chi^{(3)}$ is given in esu, β_1 in $\mathrm{s\,m^{-1}}$, and β_2 in $\mathrm{s^2\,m^{-1}}$.See text for details.

shift is limited by the properties of the control field. Note first that, if the control field intersects the signal beam at right angles, there is no reason why absorption of the control field need limit the interaction path length of the signal field. In the more common situation in which the signal and control fields are nearly copropagating, the interaction path length is limited to approximately the effective Beer's length of the control field, which for the conditions stated above is of the order of 25 mm. If we assume that the signal field is ten times weaker than the control field, we find that the nonlinear phase shift experienced by the signal field is of the order of 5 rad. We note that this value is comparable to the value of 4 rad predicted by Blow, Loudon, and Phoenix [19] for maximum squeezing by self-phase modulation in propagation through a nonlinear optical material.

Previous attempts [23] to produce squeezed light by self-phase modulation in an atomic vapor have failed to produce positive results, presumably because of the additional noise acquired by a beam in passing through an atomic vapor as the result of various atomic noise sources [23,24]. There is good reason to believe that certain of these noise sources will be greatly reduced by the EIT process. For instance, the reduction in the level of squeezing resulting from absorption of the signal beam will be entirely eliminated by EIT. Other noise sources, such as spontaneous emission following the transfer of population to the upper atomic level, will not be suppressed and will introduce noise at specific frequencies determined by the energy-level structure of the dressed atom. More detailed theoretical and experimental work needs to be performed to determine the resulting quantum noise properties of the two-level atom in the presence of the control field. It is worth noting that detailed theoretical analysis confirms the complete suppression of quantum noise in one particular EIT situation [7].

We next consider the possibility of using EIT in the two-level atom to support the propagation of slow-light spatiotemporal solitons. Spatiotemporal solitons can exist if the material possesses the following characteristics: (1) $\mathrm{Im}\,\chi^{(1)}=0$, implying lossless propagation, (2) $\mathrm{Re}\,\chi^{(3)}>0$, implying the possibility of self-trapping, and (3) β_2 and $\mathrm{Re}\,\chi^{(3)}$ have opposite signs, allowing the possibility of a nonlinear-Schrödinger-equation temporal soliton. The conditions assumed for the example given in Fig. 4 are seen also to present appropriate conditions for spatiotemporal solitons. In this case the group index n_g is approximately equal to 5. It should be noted that the existence of conditions that satisfy the three conditions listed above does not in itself guarantee that optical pulses will possess the special stability requirements needed for true solitonic propagation. The question of the stability of solitons excited under the conditions
described above remains an interesting unanswered question.

Further interesting behavior is illustrated in Fig. 5, which shows a region near the three-photon resonance. For this example we assume the same sodium and argon number densities as in Fig. 4, but a larger control laser intensity of 2 $\mathrm{MW/cm^2}$, and a larger detuning of the control laser from the atomic resonance, 20 GHz. We see that $\mathrm{Im}\,\chi^{(1)}$ is negative, implying that the signal wave experiences gain, and that β_1, β_2, and $\mathrm{Re}\,\chi^{(3)}$ change sign as the signal frequency is tuned through the resonance, thus allowing the existence of positive and negative group velocities and dark or bright spatial and/or temporal solitons. In this case the group index can be as large as 90.

In summary, we have shown that electromagnetically induced transparency can be observed in the response of the two-level atom and can lead to large self-action effects and to vanishing material absorption. These effects should be observable in realistic laboratory experiments, with important potential consequences for interactions such as squeezed-light generation and the propagation of optical solitons.

This work was supported by ONR Grant No. N00014-99-1-0539.

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