Nonlinear effects in Silicon Waveguides

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Silicon Photonics

- Silicon dominates microelectronics industry totally.
- Silicon photonics is a new research area trying to capitalize on the huge investment by the microelectronics industry.
- It has the potential for providing a monolithically integrated optoelectronic platform on a silicon chip.

Credit: Intel and IBM Websites
SOI Waveguides (Photonic Wires)

- Silicon-on-insulator (SOI) technology is used to make silicon waveguides almost routinely.
- Nonlinear effects exploited to make novel devices such as Raman amplifiers, wavelength converters, and optical switches.
- Nonlinear silicon photonics is an emerging new field.

- SEM image of an SOI device; Izhaky et al., JSTQE, (2006).
- Typically $W \sim 1$ $\mu$m, $H \sim 1$ $\mu$m, $h \sim H/2$.
- Dispersion normal at telecom wavelengths ($\lambda_0 \sim 2.2$ $\mu$m).
Dispersion Tailoring

- Mode index $\tilde{n}(\omega) = n_m(\omega) - \delta n_W(\omega)$.
- Waveguide dispersion results from $\delta n_W(\omega)$.
- Total dispersion $D = D_M + D_W$ can be tailored by controlling waveguide dimensions to below 1 $\mu$m.
Third-order Nonlinear Response

$$\tilde{P}_i^{(3)}(\omega_i) = \frac{3\varepsilon_0}{16\pi^2} \int \int \chi_{ijkl}^{(3)}(-\omega_i; \omega_j, -\omega_k, \omega_l) \tilde{E}_j(\omega_j) \tilde{E}_k^*(\omega_k) \tilde{E}_l(\omega_l) d\omega_j d\omega_k$$

- Two distinct contributions from electronic and Raman responses:
  $$\chi_{ijkl}^{(3)} = \chi_{ijkl}^e + \chi_{ijkl}^R.$$

- Electronic response for silicon is governed by
  $$\chi_{ijkl}^e = \chi_{1122}^e \delta_{ij} \delta_{kl} + \chi_{1212}^e \delta_{ik} \delta_{jl} + \chi_{1221}^e \delta_{il} \delta_{jk} + \chi_d^e \delta_{ijkl}.$$

- Raman response for silicon is governed by
  $$\chi_{ijkl}^R = \chi_R^\tilde{H}_R(\omega_l - \omega_k)(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - 2\delta_{ijkl}) + \text{a similar term}.$$

- A relatively narrow Lorentzian Raman-gain spectrum:
  $$\tilde{H}_R(\Omega) = \frac{\Omega_R^2}{\Omega_R^2 - \Omega^2 - 2i\Gamma_R\Omega} \left( \frac{\Omega_R}{2\pi} = 15.6 \text{ THz}, \frac{\Gamma_R}{\pi} = 105 \text{ GHz} \right).$$
Third-order Nonlinear Susceptibility

- The tensorial nature of $\chi^{(3)}$ makes theory quite complicated.
- It can be simplified considerably when a single optical beam excites the fundamental TE or TM mode of the Si waveguide.
- Only the component $\chi_{1111}^e(-\omega; \omega, -\omega, \omega)$ is relevant in this case.
- Its real and imaginary part provide the Kerr coefficient $n_2$ and the TPA coefficient $\beta_T$ as

$$\frac{\omega}{c} n_2(\omega) + \frac{i}{2} \beta_{\text{TPA}}(\omega) = \frac{3\omega}{4\varepsilon_0 c^2 n_0^2} \chi_{1111}^e(-\omega; \omega, -\omega, \omega).$$

- A recent review provides more details:

- We measured $n_2$ and $\beta_T$ for silicon in the spectral range of 1.2 to 2.4 $\mu$m using the $z$ scan technique.
- $n_2$ larger for silicon by $>100$ compared with silica fibers.
- TPA included using $n(\omega, I) = \bar{n}(\omega) + n_2(1 + ir)I(t)$.
- Dimensionless parameter $r = \beta_{\text{TPA}}/(2k_0n_2) \approx 0.1$ near 1.5 $\mu$m.
Free-Carrier Generation

- TPA creates free carriers inside a silicon waveguide according to
  \[
  \frac{\partial N_c}{\partial t} = \frac{\beta_{\text{TPA}}}{2h\nu_0} I^2(z, t) - \frac{N_c}{\tau_c}.
  \]

- Carrier lifetime is relatively large for silicon ($\tau_c > 10$ ns).
- It limits the device response time if carriers cannot be removed quickly enough.
- Free carriers also introduce loss and change the refractive index.
- Pulse propagation inside silicon waveguides is governed by
  \[
  \frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = ik_0 n_2 (1 + ir)|A|^2 A - \frac{\sigma}{2} (1 + i\mu) N_c A - \frac{\alpha_l}{2} A.
  \]

- $\sigma \approx 1.45 \times 10^{-21}$ m$^2$ and $\mu \approx 7.5$ near 1550 nm.
Impact of Free Carriers

- Loss induced by FCA: $\alpha_f = \sigma N_c$ with $\sigma = 1.45 \times 10^{-21} \text{ m}^2$.
- Free carriers also change the refractive index by $\Delta n = -\left(\frac{\mu}{2k_0}\right)\sigma N_c$ (free-carrier dispersion).
- This change is opposite to the index change $n_2I$ resulting from the Kerr effect.
- Parameter $\mu$ is known as the “linewidth enhancement factor” in the context of semiconductor lasers.
- Its value for silicon is close to 7.5 in the spectral region near 1550 nm.
- Absorption and index changes resulting from free carriers affect the performance of silicon waveguides.
- Quick removal of carriers helps (e.g., by applying a dc electric field).
Self-Phase Modulation and TPA

• TPA reduces the maximum phase shift:
  \[ \phi_0 = \ln(1 + 2r\phi_{\text{max}})/(2r) \]

• In the absence of TPA, \[ \phi_0 = \phi_{\text{max}} = \gamma P_0 L_{\text{eff}}. \]

• Inset shows the reduction using \( r = 0.1 \).

• TPA-induced reduction becomes severe at high powers.

• When \( \phi_{\text{max}} = 100 \), \( \phi_0 \) is limited to a value of 15.
Impact of Free-Carrier Generation

Free carriers produce a nonlinear phase shift in the opposite direction.

\[
P(t) = P_0 e^{-t^2/T_0^2}
\]

\[
T_0 = 10 \text{ ps}
\]

\[
L = 2 \text{ cm}
\]

\[
\tau_c = 1 \text{ ns}
\]

\[
\alpha_l = 1 \text{ dB/cm}
\]

3. Experimental results and discussion

3.1 Nonlinear-index coefficient $n_2$

Figure 1 shows a series of TE-transmission spectra of the SOI waveguide for different coupled peak powers $P$. The two lowest power spectra with $P = 1.8$ and 8 mW, (corresponding to peak intensities of $I \sim 0.003$ and 0.014 GW/cm$^2$, respectively) only differ in the transmitted intensity but not in spectral shape and position. The spectral full width at half maximum (FWHM) is identical to pulses measured without sample. Hence, the applied powers of the two low-power spectra belong to the regime where the waveguide is responding optically linear. However, by increasing the peak power to $P = 1.55$ W ($I \approx 2.6$ GW/cm$^2$), the spectral shape of the pulses broadens dramatically; now exhibiting several spectral side wings. The broadening continues for higher powers which can be seen in the spectrum with $P = 6.85$ W ($I \approx 11.6$ GW/cm$^2$).

The measured spectral pulse distortions can be explained by self-phase-modulation, which is known to cause an intensity dependent phase shift of the pulse carrier frequency. Within its own duration, the pulse experiences an intensity- and thus time-dependent refractive index. New frequency components are generated and the initial pulse spectrum broadens in an oscillatory manner while the temporal pulse shape remains unaffected. The degree of SPM-induced spectral broadening depends on the nonlinear refractive index $n_2$ and the waveguide length $L$. Since the number of spectral oscillations in the transmission spectra is directly correlated with the nonlinear phase shift $\Phi$, we can estimate $n_2$ from the experiment by applying the formula

$$n_2 = \frac{\Phi \cdot c \cdot A_{\text{eff}}}{P \cdot L_{\text{eff}} \cdot \omega} \ [22].$$

$L_{\text{eff}}$ corresponds to an effective length of the waveguide that is smaller than $L$ because of intrinsic propagation losses.

Assuming a homogeneous and isotropic medium with cubic optical nonlinearity the relation between nonlinear phase shift $\Phi$ and number of peaks $N$ in the SPM-broadened spectra is given by

$$\Phi \approx (N - 0.5) \pi \ [22].$$

However, in the spectra presented in Fig. 1, the peaks are not very pronounced indicating that the applied coupled powers do not cause odd multiples of $0.5 \pi$ phase shifts but rather intermediate values. Hence, the visibility of the SPM-induced fringes (self-interference) is reduced making the determination of $N$ difficult.

We have therefore applied large error bars allowing the interpretation of either one large or four small peaks (see green arrows in Fig. 1 for 1.55 W). The nonlinear phase shift for the two highest coupled powers then yields $\Phi = 2 \pm 1.5 \pi$ and $\Phi = 3 \pm 1.5 \pi$ (for 1.55 W and 6.85 W, respectively) resulting in an average nonlinear-index coefficient of about $n_2 = (5 \pm 4) \cdot 10^{-18}$.

Wavelength Dependence of SPM

The results in Figs. 1 - 3 demonstrate that nonlinearities such as SPM and TPA-induced FCA strongly perturb the spectral shape of optical pulses at 1500 nm. However, telecommunication bands cover spectral ranges between 1490 nm and 1612 nm (S, C, and L-band). It is therefore of importance to have knowledge about the SPM wavelength dependence.

In order to study this in detail we have measured the spectral broadening as a function of wavelength (1400 – 1650 nm) while keeping coupled power and pulse width constant. Figure 4 compares transmission spectra at 1400 nm and 1650 nm with the spectrum at 1500 nm (same as in Fig. 1). The peak power is set to 6.85 W. At all excitation wavelengths the output spectra are broadened and asymmetric. The significant difference is that SPM-induced broadening becomes more efficient for longer wavelengths. The spectral distance between the short-wavelength SPM oscillation and the originally injected laser wavelength increases from 8 to 16 nm when the OPA is tuned from 1400 to 1650 nm. As will be discussed in the next section, this enhancement for longer wavelengths is probably due to an increase of group index \( n_g \) and nonlinear refractive index \( n_2 \).

### 4. Comparison with theory and discussion

#### 4.1 Model description

To analyze the experimental data we have used a recently developed theoretical model to describe the pulse dynamics in Si photonic wires. It accounts for GVD, parametric and non-parametric nonlinear effects such as SPM, XPM, TPA, or Raman interaction; free carrier-induced effects, such as FCA and free carriers dynamics [31]. In the case studied here, of single-frequency pulse propagation, a simplified version of the model, excluding XPM and Raman terms, is used. The dynamics of pulse propagation in silicon photonic wires is governed by the following system of coupled nonlinear differential equations:

\[
\begin{align*}
\frac{\partial A}{\partial z} & = - \beta_2 A - \alpha A^2 \psi + \delta \beta_0 \frac{\partial^2 \psi}{\partial t^2} \\
\frac{\partial \psi}{\partial t} & = - \beta_2 \psi - \alpha \psi A^2 - \delta \beta_0 \frac{\partial^2 A}{\partial z^2} \\
\frac{\partial \epsilon}{\partial t} & = \Gamma \left( \epsilon - \epsilon_0 \right)
\end{align*}
\]

Fig. 4. SPM-induced spectral broadening of optical pulses for the same coupled peak power (6.85 W) injected into the SOI waveguide at 1400, 1500 and 1600 nm. The spectral broadening increases with the wavelength.

- SPM-broadened spectra at three different wavelengths.
- 1.8-ps pulses tunable from 1400 to 1650 nm.
- Larger broadening at longer wavelengths.
- Consistent with a larger \( n_2 \) near 1550 nm

- SPM data agree with our experimental measurements of \( n_2 \) and other theoretical predictions.
Formation of Optical Solitons

- SPM-induced phase shift $\phi_{NL} > 1$ is easily realized.
- $\phi_{NL} = 1$ occurs at $z = L_{NL} = 1/(\gamma P_0)$.
- Nonlinear length $L_{NL} \sim 1$ cm at moderate peak powers $<10$ W.
- Dispersion length ($L_D = T_0^2/|\beta_2|$) can also be made $\sim 1$ cm for pulses 100-fs wide or less.
- Pulses propagate as fundamental solitons ($N = 1$) when
  \[ N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} = 1. \]
- Formation of optical solitons possible if waveguide is designed to provide anomalous dispersion at the operating wavelength.
- Numerical simulations confirm this expectation.
Formation of Optical Solitons

130-fs pulses launched inside a 5-mm-long waveguide ($N = 1$).
Soliton-Induced Spectral Narrowing

- In a 2007 experiment [Opt. Exp. 15, 7682 (2007)], we launched 110-fs Gaussian pulses inside a 5-mm-long waveguide.
- Gaussian spectrum broadened at 1250 nm because of $\beta_2 > 0$.
- It narrowed and acquired a “sech” shape at 1484 nm where $\beta_2 < 0$. 
Supercontinuum Generation

- Ultrashort pulses are affected by a multitude of nonlinear effects, such as SPM, XPM, FWM, and SRS, together with dispersion.

- All of these nonlinear processes are capable of generating new frequencies outside the input pulse spectrum.

- For sufficiently intense pulses, the pulse spectrum can become so broad that it extends over a frequency range exceeding 100 THz.

- Such extreme spectral broadening is referred to as supercontinuum generation.

- This phenomenon was first observed in solid and gases more than 35 years ago (late 1960s.)

- Since 2000, microstructure fibers have been used for supercontinuum generation.
SC Generation in Silicon Waveguides

50-fs pulses launched with 25-W peak power into a 1.2-cm waveguide.
Impact of TPA on SC Generation


- TPA reduces SC bandwidth but is not detrimental.
- Nearly 400-nm-wide supercontinuum created within a 3-mm-long waveguide.
- Required pulse energies are relatively modest (\(\sim 1\) pJ).
Cross-Phase Modulation

- Consider two optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on the intensity of the other wave as
  \[ \Delta n_{NL} = n_2(|A_1|^2 + b|A_2|^2). \]
- Total nonlinear phase shift in a fiber of length \( L \):
  \[ \phi_{NL} = \frac{2\pi L}{\lambda} n_2[I_1(t) + bI_2(t)]. \]
- An optical beam modifies not only its own phase but also of other copropagating beams (XPM).
- XPM induces nonlinear coupling among overlapping optical pulses.
XPM-Induced Spectral Changes


- 200-fs pump and probe pulses (at 1527 and 1590 nm) launched into a 4.7-mm-long SOI waveguide ($w = 445$ nm, $h = 220$ nm).
- Pump and probe pulses travel at different speeds (walk-off effect).
- XPM-induced phase shifts occur as long as pulses overlap.
- Asymmetric XPM-induced spectral broadening depends on pump power (blue curve); Probe spectra without pump (red curve).
XPM-Induced Switching


- A Mach–Zehnder interferometer used for optical switching.
- Short pump pulses (<1 ps) at 1560 nm pass through the arm containing a 2.5-cm-long SOI waveguide.
- CW probe experiences XPM-induced phase shift in that arm.
- Temporal slice of the probe overlapping with the pump is optically switched.
Four-Wave Mixing (FWM)

- FWM is a nonlinear process that transfers energy from pumps to signal and idler waves.
- FWM requires conservation of
  - Energy \( \omega_1 + \omega_2 = \omega_3 + \omega_4 \)
  - Momentum \( \beta_1 + \beta_2 = \beta_3 + \beta_4 \)
- Degenerate FWM: Single pump \( (\omega_1 = \omega_2) \).
FWM Theory for Silicon Waveguides

- Theory should include TPA and free-carrier effects fully.
- Polarization and Raman effects should also be included.
- Free-carrier absorption limits the gain for a CW pump.
- \( \beta_2 < 0 \) (red); \( \beta_2 = 0 \) (blue); \( \beta_2 > 0 \) (green).
FWM with Short Pump Pulses


- FCA is reduced significantly for pump pulses much shorter than carrier lifetime $\tau_c$.
- Figure shows the case of 10-ps pump pulses with $\tau_c = 1$ ns.
- Phase-matching condition is satisfied even for signal that is shifted by 70 nm from the pump wavelength.
Singe and Dual-Pump Configurations


- Parametric amplifiers with a large bandwidth can be realized by pumping an SOI waveguide with two pumps.
- This is possible because of a relatively short device length.
- Recent experiments with SOI waveguides are encouraging.
Experimental Results (Single Pump)


- 3.5-ps pump pulses at 1550 nm; signal tunable over 80 nm.
- Up to 5 dB of gain observed in devices <2 cm long.
Photon-Pair Generation


- Spontaneous FWM in fibers creates entangled photon pairs but suffers from the noise induced by Raman scattering.
- The use of SOI waveguides avoids this problem because Raman scattering does not occur when TM mode is excited.
Experimental Results


- 5-ps pump pulses launched into a 9-mm-long SOI waveguide.
- Total (red) and accidental (blue) coincidence rates measured.
- Their ratio exceeded 10 at low pump powers.
Raman Amplifiers

- CW pumping leads to accumulation of free carriers through TPA.
- Free-carrier absorption introduces losses for pump and signal.
- No signal gain occurs for $\tau_{\text{eff}} > 10$ ns.

Jalali et al., IEEE JSTQE 12, 412 (2006)
CW Raman Amplifiers


- CW pumping can be used if free carriers are removed quickly.
- A reversed-biased p-n junction is used for this purpose.
- Electric field across the waveguide removes electrons and holes.
- Drift time of carriers is shorter for larger applied voltages.

- A 4.8-cm-long waveguide CW pumped at 1458 nm (signal at 1684 nm).
- Output pump and signal powers increase with applied voltage.
- Effective carrier lifetime decreases from 16 to 1 ns.
Concluding Remarks

- Nonlinear effects in silicon waveguides can be used to make many active and passive components.

- SPM is useful for soliton formation and supercontinuum generation.

- SPM and XPM can also be used for optical switching, wavelength conversion, and all-optical regeneration.

- Four-wave mixing converts silicon waveguides into parametric amplifiers.

- It can also be used for quantum applications requiring entangled photon pairs.

- Stimulated Raman scattering converts silicon waveguides into Raman amplifiers.